1 Introduction

Fluctuations in terms of trade have been considered as an important source in generating volatility and business cycles in open and relatively small economies, where prices of import and export goods are exogenously given in the world market and pass-through rate is relatively high. The question of how should monetary authority in a small open economy should respond to changes in terms of trade, therefore, has been an important topic in the literature.

This paper revisits the classical question in the framework of the so-called New Open Economy Macroeconomics, henceforth NOEM.[1] In particular, we studies a small open economy model, where households have market power in wage setting because labor is differentiated across households. Moreover, we consider two types of wage setting: flexible wage-setting and predetermined wage-setting; in the former environment households are free to set wages while in the latter households have to set wages in in advance.

In addition, unlike most literature in welfare analysis of optimal monetary policy in which linear-quadratic and second order approximation are applied, [2] a peculiar point of this paper is the application of Ramsey-type analysis for optimal monetary policy in small open economy. Specifically, in this Ramsey type approach, the Ramsey planner (in this paper, the monetary authority) maximizes the household's welfare subject to the constraints that characterize the equilibrium in the private sector, the resource constraint, and characteristics of the economy. The Ramsey approach has been applied in the study of optimal policy in dynamic models of closed economics: for instance, Chari and Kehoe (1999) with flexible prices, Adao et al (2003) and Khan et al (2003) with monopolistic competition and nominal rigidities.

A Ramsey-type approach also has been studied in the context of New Keynesian small open economy models with the work of Faia and Mona-

[1]This literature has grown very fast after the seminal work of Obstfeld and Rogoff (1996). See for instance Lane, P. (2000) and Sarno, L. (2001) for surveys.

[2]This approach is built on the work of Rotemberg and Woodford (1997)

celli (2006).[3] In this paper, it is shown that home bias in consumption is a sufficient condition for inducing monetary policy makers of small open economy to deviate from strict markup stabilization and have some degree of exchange rate stabilization. The major difference between this paper and Faia and Monacelli (2006) is that we consider real balances in utility function and nominal rigidity and distortions from the labor market. With some specifications in the utility function, we are able to solve for the optimal level of money supply as a function of wage and exogenous terms of trade and conduct analytical comparison.

The findings of this paper is as follows: under the free wage-setting environment, monetary policy can not affect the real economy or money is neutral in this environment as usual. By contrast, the predetermined wage environment enables the monetary authority to impact the real economy, hence welfare could be improved upon a distorted flexible wage allocations; and the optimal welfare does not depend on labor market friction conditions. Nonetheless, the way to implement the optimal monetary policy response depends crucially on the value of elasticity of substitution between tradable goods and nontradable goods. Optimal money supply increases (decreases) with a fall in terms of trade when the elasticity of substitution is greater (less) than one. Moreover, for the same terms of trade, the welfare of households is always higher when the value of elasticity of substitution between tradable goods and nontradable goods are greater.

However, under a special case where labor supply of household is very elastic [4], we find that the Ramsey optimal money supply is independent from terms of trade regardless of the value of elasticity of substitution. In other words, at the optimal point, the monetary authority utilizes the predetermined wage environment and supplies an extra amount of money to undo the distortion caused by labor market[5] and let the goods markets solve for

[3] In the context of two country model, see Faia and Monacelli (2004)

[4] Labor dis-utility enters utility function as linear form, which can be interpreted as indivisible labor

[5] Actually, monetary authority can do more than undo the labor market distortion by

efficient allocations in response to fluctuations in terms of trade.

The structure of this paper is as follows: section 2 develops a small open economy model with labor market frictions; section 3 discusses the flexible wage-setting equilibrium while section 4 analyses the optimal monetary policy under the predetermined wage environment. Section 5 concludes the paper.

2 A small open economy model

We consider a small open economy model with tradable sector and non-tradable sector. The economy is small and open in the sense that it takes the foreign currency prices of tradable goods and imported intermediate goods as given. It is a New Keynesian model as the labor market is demand-driven and wage is predetermined.

In addition, we consider an economy where the government (in particular the monetary authority in this paper) behaves optimally to maximize the household's welfare as the Ramsey approach. That is the Ramsey type monetary authority maximizes household's welfare subject to the resource constraint and to the constraints that describe the equilibrium in the private sector economy and the predetermined wage constraint.

2.1 Firms-Productions

2.1.1 Non-tradable Goods Sector

Non-tradable goods are produced by a technology that is the Cobb-Douglas function of composite labor and imported intermediate goods (e.g. oil) as follows:

$$Y_N = AL^{\alpha}I^{1-\alpha} \tag{1}$$

take advantage of real balances in utility function to improve household's welfare as later shown in the paper where I is the amount of imported intermediate goods with foreign currency price P_I exogenously given to this small open economy. Labor is differentiated across households, which allows households to have market power in wage setting. Composite labor is defined as follows:

$$L_t \equiv \Big(\int_0^1 L_t(i)^{\frac{\phi-1}{\phi}} di \Big)^{\frac{\phi}{\phi-1}} \tag{2}$$

where $L(i)$ is labor supplied by the household i, and $\phi > 1$ is the elasticity of substitution between labor varieties; the higher ϕ is the more competitive labor market is. As a result, the wage index is defined as:

$$W_t \equiv \Big(\int_0^1 W_t(i)^{1-\phi} di \Big)^{\frac{1}{1-\phi}} \tag{3}$$

where $W(i)$ is the nominal wage set by the household i.

Therefore, profits of a non-tradable goods firm are:

$$\Pi = P_N Y_N - WL - SP_I I \tag{4}$$

Non-tradable firms maximize profits taking the nominal wage W and the foreign currency price of intermediate goods P_I as given and S is the nominal exchange rate.

Non-tradable firms choose labor to maximizes profits, which the gives us the implicit labor demand as follows:

$$W(i) = \alpha \frac{Y_N}{L} \Big(\frac{L(i)}{L} \Big)^{\frac{-1}{\phi}} P_N \tag{5}$$

Under the competition market assumption about nontraded goods with free entry, the price of nontraded goods is determined as:

$$P_N = \kappa \frac{W^{\alpha} (SP_I)^{1-\alpha}}{A} \tag{6}$$

where $\kappa \equiv (\frac{1}{1-\alpha})^{1-\alpha} (\frac{1}{\alpha})^{\alpha}$

2.1.2 Tradable goods

Each household in this small open economy is endowed with a fixed amount y_T of tradable goods and the price of these tradable goods is exogenously determined in the international markets. We normalize the foreign-currency price of tradable goods to unity hence the domestic price of tradable goods is equal to the nominal exchange rate S and the inverse terms of trade is equal to P_I.

2.2 Households

The economy consists of total measure unity households, each household has the same preference. Real balances enter into the household's preference and the composite consumption index is a CES function of tradable goods and nontradable goods where the elasticity of substitution between the two types of goods might be different from unity.

The utility function of the household $i, i \in [0,1]$ has the following form:

$$U(i) \equiv ln(C(i)) + \chi ln\Big(\frac{M(i)}{P}\Big) - \frac{\eta}{2}L(i)^2 \qquad (7)$$

where $C(i)$ is the composite consumption of tradables and non-tradables, defined by a CES form as follows:

$$C \equiv \left[(1-\gamma)^{\frac{1}{\theta}}(C_T)^{\frac{\theta-1}{\theta}} + (\gamma)^{\frac{1}{\theta}}(C_N)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \qquad (8)$$

where θ is the elasticity of substitution between tradable and nontradable goods and γ represents the relative preference of nontradable goods. $M(i)$ is the quantity of real balances or the amount of domestic money held by the household i. The consumer price index (CPI) P, therefore, can be expressed as:

$$P \equiv \left[(1-\gamma)(P_T)^{1-\theta} + (\gamma)(P_N)^{1-\theta}\right]^{\frac{1}{1-\theta}} \qquad (9)$$

The household, i, faces the following budget constraint:

$$PC(i) + M(i) = W(i)L(i) + M_0(i) + \tau + Sy_T + \Pi(i) \qquad (10)$$

where $M_0(i)$ is the initial amount of real balances holdings, τ is the total transfer from the government (the monetary authority), and $\Pi(i)$ denotes the profits obtained from nontradable good firms.[6]

The optimality of the household i implies:

$$M(i) = \chi PC(i) \tag{11}$$

$$C_T(i) = (1-\gamma)(\frac{P_T}{P})^{-\theta}C(i) \tag{12}$$

$$C_N(i) = \gamma(\frac{P_N}{P})^{-\theta}C(i) \tag{13}$$

The household i faces a downward sloping labor demand curve given in equation (5), therefore, the nominal wage of a monopolistic labor supply depends on whether wage is freely and simultaneously set or predetermined.

If wage is flexibly set, the optimal wage is chosen as follows:

$$W(i) = \eta\frac{\phi}{\phi-1}L(i)PC(i) = \eta\frac{\phi}{\phi-1}L(i)\frac{M(i)}{\chi} \tag{14}$$

While if wage is predetermined or set in advance as W^p then, the equation (5) determines labor supply.

2.3 Monetary Authority

The monetary authority in this small open economy chooses the level of money supply M after observing terms of trade P_l and wage, W^p, if it is predetermined or set in advance. In the meantime, the monetary authority also realizes wage is freely and simultaneously set, otherwise. In either cases, the monetary authority transfers all gains from issuing money equally to all households as follows:

$$\tau = M(i) - M_0(i) \tag{15}$$

[6] I assume that each measure of household get the same amount of profits.

2.4 Equilibrium

We focus on the symmetric equilibrium, in which:

$$C(i) = C, W(i) = W, L(i) = L, M(i) = M, M_0(i) = M_0, \forall i \in [0, 1] \quad (16)$$

Then the wage equation (5) implies

$$W = \alpha \frac{Y_N P_N}{L} \Leftrightarrow WL = \alpha Y_N P_N \quad (17)$$

Money market clearing:

$$PC = \frac{M}{\chi} \quad (18)$$

Nontradable goods market clearing:

$$C_N = Y_N \quad (19)$$

These conditions together with (10) imply:

$$WL = \alpha\gamma(\frac{P_N}{P})^{1-\theta} PC = \alpha\gamma(\frac{P_N}{P})^{1-\theta} \frac{M}{\chi} \quad (20)$$

The household budget constraint becomes:

$$PC = WL + Sy_T \quad (21)$$

We, now, proceed to consider the how wages are determined.

3 Flexible Wage Setting

First, we consider the case where wages are flexible or households are able to set wage freely and simultaneously subject to the given downward sloping demand for labor in (5). The optimal wage becomes:

$$W_f = \frac{\eta\phi}{\chi(\phi - 1)} ML \quad (22)$$

Notice that the elasticity of substitution between labor varieties ϕ enters into the wage determining equation; the higher ϕ is (more competitive between labor market), the lower the nominal wage is.

Hence, the equations that describes the equilibrium in this flexible wage setting small open economy are as follows:

$$PC = \frac{M}{\chi} \tag{12}$$

$$WL = \alpha\gamma(\frac{P_N}{P})^{1-\theta}\frac{M}{\chi} \tag{16}$$

$$PC = WL + Sy_T \tag{17}$$

$$W = W_f = \frac{\eta\phi}{\chi(\phi - 1)}ML \tag{18}$$

Since CPI can be expressed as:

$$P = S[(1-\gamma) + \gamma(\frac{P_N}{S})^{1-\theta}]^{\frac{1}{1-\theta}} \tag{23}$$

$$= P_N[(\gamma + (1-\gamma)(\frac{S}{P_N})^{1-\theta}]^{\frac{1}{1-\theta}} \quad \text{and;} \tag{24}$$

$$(\frac{S}{P_N})^{1-\theta} = \left[\frac{A}{\kappa}(\frac{S}{W})^{\alpha}(\frac{1}{P_I})^{1-\alpha}\right]^{1-\theta} \tag{25}$$

In other words, P, $\frac{P_N}{P}$ are also a function of the nominal exchange rate S and wage level W, respectively. Therefore, for a given level of money supply M and terms of trade P_I, equations (12), (16), (17), (18) will determine consumption, labor, wage, and the nominal exchange rate C, L, W, S, respectively.

Proposition 1 *Under the flexible wage setting environment, money is neutral in this small open economy, i.e., the monetary authority can not affect the households' welfare, which is however impacted by the terms of trade fluctuations.*

Proof: First, substitute (12), (18) into (17), we obtain:

$$\frac{M}{\chi} = \frac{W^2}{\psi M} + S y_T$$
$$W^2 = M^2\Big(\frac{\psi}{\chi} - y_T\frac{S}{M}\Big)$$
$$W = M\Big(\frac{\psi}{\chi} - \psi y_T\frac{S}{M}\Big)^{1/2} \tag{26}$$

where $\psi \equiv \frac{\eta\phi}{\chi(\phi-1)}$

Second, substitute (12), (16), (20), (21) into (17) and rewrite:

$$\left[1 - \frac{\alpha\gamma}{\gamma + (1-\gamma)\Big[\frac{A}{\kappa}(\frac{S}{W})^{\alpha}(\frac{1}{P_I})^{1-\alpha}\Big]^{1-\theta}}\right] = \chi y_T \frac{S}{M} \tag{27}$$

Finally, substitute wage W from (22) into (23) we obtain a equation for ratio $\frac{S}{M}$ as:

$$\left[1 - \frac{\alpha\gamma}{\gamma + (1-\gamma)\Big[\frac{A}{\kappa}(\frac{S}{M})^{\alpha}\Big(\frac{\psi}{\chi} - \psi y_T\frac{S}{M}\Big)^{-\frac{1}{2}\alpha}(\frac{1}{P_I})^{1-\alpha}\Big]^{1-\theta}}\right] = \chi y_T \frac{S}{M} \tag{28}$$

which implies that the ratio $\frac{S}{M}$ is always pinned down by the exogenously given intermediate goods price P_I. A a result, when wage are flexibly set, the nominal exchange rate S is always proportional to the money supply level M in equilibrium.

Furthermore, the utility of houschold can be written as follows:

$$U = ln(C) + \chi ln\Big(\frac{M}{P}\Big) - \frac{\eta}{2}L^2 = (1+\chi)ln\Big(\frac{M}{P}\Big) - \frac{\eta}{2}L^2 \tag{29}$$

From (19), (21), (22), the log of real balances $ln(\frac{M}{P})$ can be expressed as:

$$\begin{aligned} ln(\frac{M}{P}) &= ln\Big(\frac{M}{S}\Big) - \frac{1}{1-\theta}ln\Big(1-\gamma+\gamma(\frac{P_N}{S})^{1-\theta}\Big) \\ &= ln\Big(\frac{M}{S}\Big) - \frac{1}{1-\theta}ln\Big(1-\gamma+\gamma\Big[\frac{A}{\kappa}\Big(\frac{\psi}{\chi} - \psi y_T\frac{S}{M}\Big)^{\frac{-\alpha}{2}}(\frac{S}{M})^{\alpha}(\frac{1}{P_I})^{1-\alpha}\Big]^{\theta-1}\Big) \end{aligned} \tag{30}$$

In addition, in this environment, from (18), (22), the employment can be expressed as:

$$L = \frac{W}{\psi M} = \frac{1}{\psi}\Big(\frac{\psi}{\chi} - \psi y_T \frac{S}{M}\Big)^{1/2} \tag{31}$$

Equations (25), (26), (27) implies that household's utility or welfare is a function of $\frac{S}{M}$, P_I, hence, is only a function of exogenous variable P_I since it is shown above that $\frac{S}{M}$ is a function of P_I.

Therefore, we can conclude that under the flexible wage setting environment, monetary authority can not affect the utility of household, i.e., money is neutral for this small open economy. The households' welfare in completely affected by the movement of exogenous terms of trade P_I.

Notice that the household welfare depends on parameter ϕ that represents the monopoly power of labor supply. The more competitive the labor market is (higher ϕ) the higher household welfare is.

Although for a given money supply M, the CPI always increases when the inverse terms of trade P_I goes up, equation (24) implies that the nominal exchange rate S can appreciate, depreciate or stay constant for $\theta > 1$, $\theta < 1$, or $\theta = 1$, respectively. Intuitively, when tradable goods and non-tradable goods are substitute ($\theta > 1$), an increase in imported intermediate good price P_I induces higher nontradable goods prices, which makes consumers demand more tradable goods, hence, leads to an increase in domestic currency price of tradable goods that is the same as nominal exchange rate in our model S. By contrast, when nontradable goods and tradable good are complement, i.e., $\theta < 1$, an increase in nontradable goods prices induced from an increase in imported intermediate goods foreign currency price leads to less demands on on both nontradable and tradable goods, hence, nominal exchange rate or domestic currency price of traded goods S falls. When the nominal exchange rate S falls, the domestic currency price of intermediate goods SP_I becomes less expensive and when $\theta = 1$, the two effects cancel each other out so that eventually nominal exchange rate S unchanged.

Figure 1 and 2 show the responses of the nominal exchange rate S, CPI P, composite consumption C, and employment when there are fluctuations in foreign currency price of intermediate goods P_I under $\theta = 1.5$ and $\theta = 0.5$, respectively. It is shown that when P_I goes up, employment rises (falls) with $\theta < 1$ ($\theta > 1$). The reason is that when P_I goes up, the nominal exchange rate or domestic price of tradable goods S increases with $\theta > 1$, hence, households' increases. Since CPI rises, households can substitute consumption with leisure hence supply less labor. In contrast, if $\theta < 1$, S falls, households' income decreases, therefore when money supply is kept fixed, households need to supply more labor to compensate less income from the tradable endowment.

Figure 3 presents the response of welfare when there are fluctuations in intermediate goods price. Welfare decreases with an increase in P_I regardless of the relative value of θ to 1. However, welfare is always higher for $\theta > 1$ compared to the same P_I under $\theta < 1$ case.

4 Predetermined Wage Setting

Now, we consider the case where wages are predetermined, i.e., wages are set by households in advance. We define a (symmetric) equilibrium, given price of imported intermediate goods P_I, endowment y_T, initial money holding M_0, and a monetary rule, as the set of allocation $\{C_N, C_T, C, L, M\}$ and the set of prices $\{W^p, S, P_N, P\}$ such that:

1. Households set wage, W_p, in advance;
2. Firms maximize profits;
3. Households maximize their utility over consumption and real balances subject to ex-post budget constraints;
4. The money market clears:

$$M = M_0 + \tau \tag{32}$$

5. The nontradable goods market clears:

$$Y_N = C_N \tag{33}$$

Under this predetermined wage setting environment, equations that characterizes equilibrium are as follows:

$$PC = \frac{M}{\chi} \tag{12}$$

$$WL = \alpha\gamma(\frac{P_N}{P})^{1-\theta}\frac{M}{\chi} \tag{16}$$

$$PC = WL + Sy_T \tag{17}$$

$$W = W_p \tag{34}$$

these equations then imply

$$\left[1 - \frac{\alpha\gamma}{\gamma + (1-\gamma)\left[\frac{A}{\kappa}(\frac{S}{W_p})^{\alpha}(\frac{1}{P_I})^{1-\alpha}\right]^{1-\theta}}\right]\frac{M}{\chi} = Sy_T \tag{35}$$

With predetermined W_p, from this equation we obtain the nominal exchange rate S as a function of money supply M and the price of imported intermediate goods P_I or the inverse of terms of trade as follows:

$$S = F(M; W^p, P_I) \tag{36}$$

Then with the nominal exchange rate or price of tradable goods, S, known, we can solve for other price levels such as P_N, P; and from the money market clearing condition, we obtain the level of consumption C, which in turn gives C_T, C_N and employment L.

Proposition 2: *In a small open economy with the predetermined wage setting, money supply can affect employment and the nominal exchange rate is an increasing function of money supply.*

Proof: In the Appendix.

Therefore, unlike under the flexible wage setting environment where money is neutral, monetary policy can affect employment, consumption, hence, and the household's welfare when wages are predetermined, which is consistent with traditional Keynesian theories.

Next, we consider the Ramsey-type problem for the monetary authority, i.e., the monetary authority chooses the optimal money supply M that maximizes household's welfare subject to constraints that characterize the equilibrium in the private sector economy and the predetermined wage level. Formally, the monetary authority chooses M to maximize:

$$ln(C) + \chi ln(\frac{M}{P}) - \frac{\eta}{2}L^2 \tag{37}$$

subject to:

$$PC = \frac{M}{\chi} \tag{12}$$

$$WL = \alpha\gamma(\frac{P_N}{P})^{1-\theta}\frac{M}{\chi} \tag{16}$$

$$PC = WL + Sy_T \tag{17}$$

$$W = W^p \tag{27}$$

Proposition 3 *The optimal money supply is proportional to predetermined wage and is a function of terms of trade. However, the direction of the optimal money supply response depends on the value of θ: M is increasing (decreasing) with P_I if $\theta > 1$ ($\theta < 1$), M remains constant when $\theta = 1$.*

From (12), (17) and (27), we can express labor L as:

$$L = \frac{1}{W_p}(\frac{M}{\chi} - Sy_T) \tag{38}$$

Therefore, we can rewrite the planner's problem as choosing M to maximize

$$ln(\frac{M}{P}) - \frac{\eta}{2(1+\chi){W_p}^2}(\frac{M}{\chi} - Sy_T)^2 \tag{39}$$

subject to

$$\left[1-\frac{\alpha\gamma}{\gamma+(1-\gamma)\left[\frac{A}{\kappa}(\frac{S}{W^p})^{\alpha}(\frac{1}{P_I})^{1-\alpha}\right]^{1-\theta}}\right]M=\chi S y_T \tag{29}$$

Since,

$$P=\left[(1-\gamma)S^{1-\theta}+\gamma P_N^{1-\theta}\right]^{\frac{1}{1-\theta}}=S[(1-\gamma)+\gamma(\frac{P_N}{S})^{1-\theta}]^{\frac{1}{1-\theta}} \tag{40}$$

Let's denote

$$(\frac{S}{P_N})^{1-\theta}=\left[\frac{A}{\kappa}(\frac{S}{W^p})^{\alpha}(\frac{1}{P_I})^{1-\alpha}\right]^{1-\theta}\equiv G(S) \tag{41}$$

then

$$P=S[(1-\gamma)+\frac{\gamma}{G(S)}]^{\frac{1}{1-\theta}} \tag{42}$$

In the Appendix, we show that under optimal policy, the money supply M and the nominal exchange rate S must satisfy two following equations:

$$\frac{1}{M}=\frac{\eta}{\chi(\chi+1)W_p^2}(\frac{M}{\chi}-Sy_T) \tag{43}$$

$$M\left(1-\frac{\alpha\gamma}{\gamma+(1-\gamma)G(S)}\right)=\chi S y_T \tag{44}$$

Rewrite (34) as:

$$M^2-\chi y_T SM-\frac{\chi^2(\chi+1)}{\eta}W_p^2=0 \tag{45}$$

This gives :

$$M=\frac{1}{2}\left(\chi y_T S+\sqrt{(\chi y_T S)^2+4\frac{\chi^2(\chi+1)}{\eta}W_p^2}\right) \tag{46}$$

Substitute (32) and (37) into (35), we obtain S as function of P_I and W_p as:

$$\left[1-\frac{\alpha\gamma}{\gamma+(1-\gamma)\left[\frac{A}{\kappa}(\frac{S}{W_p})^{\alpha}(\frac{1}{P_I})^{1-\alpha}\right]^{1-\theta}}\right]\left(1+\sqrt{1+4\frac{\chi(\chi+1)}{\eta y_T^2}(\frac{W_p}{S})^2}\right)=2 \tag{47}$$

Notice that, in this case, at the optimal point, the ratio $\frac{S}{W_p}$ is completely pinned down by P_I and other parameters. Denote this function as $H(P_I)$, we can write:

$$\begin{aligned} \frac{S}{W_p} &= H(P_I) \\ S &= W_p H(P_I) \end{aligned} \tag{48}$$

Note that $H(P_I)$ is increasing (decreasing) function of P_I when $\theta > 1$ ($\theta < 1$).

Substitute (39) back to (37), we obtain the optimal monetary supply as a function of predetermined wage and P_I:

$$M = \frac{W_p}{2}\left(\chi y_T H(P_I) + \sqrt{(\chi y_T H(P_I))^2 + 4\frac{\chi^2(\chi+1)}{\eta}}\right) \tag{49}$$

From (49), it is straightforward to see that M is proportional to W_p and is an increasing (decreasing) function of P_I when $\theta > 1$ ($\theta < 1$).

Proposition 4 *At optimality, the household's welfare is independent from the predetermined wage level and the labor market friction conditions. Household's welfare is improved upon the distorted flexible wage setting allocations.*

In other words, predetermined wage setting environment provides the monetary authority effective tools to improve households' welfare in comparison with the allocations obtained under distorted flexible wage environment. Consequently, at the optimal point, household's welfare is independent from the predetermined wage level W_p and the parameter ϕ that represents the frictions of labor market.

The equation (49) shows that the ratio $\frac{S}{W_p}$ is pinned down by P_I and independent from ϕ, therefore, it is sufficient to show that the household's utility is a function of $\frac{S}{W_p}$ and P_I but is independent from ϕ.

Substituting the CPI's expression from (33), households' utility (welfare) can be rewritten as:

$$\begin{aligned} U &= (1+\chi)ln(\frac{M}{P}) - \frac{\eta}{2}L^2 \\ &= (1+\chi)\Big[ln(\frac{M}{S}) - \frac{1}{1-\theta}ln(1-\gamma+\frac{\gamma}{G(S)})\Big] - \frac{\eta}{2}L^2 \end{aligned} \tag{50}$$

From (37), we can write ratio $\frac{M}{S}$ as:

$$\frac{M}{S} = \frac{1}{2}\Bigg(\chi y_T + \sqrt{(\chi y_T)^2 + 4\frac{\chi^2(\chi+1)}{\eta}(\frac{W_p}{S})^2}\Bigg) \tag{51}$$

Recall the definition of $G(S)$ as:

$$G(S) \equiv \Big[\frac{A}{\kappa}(\frac{S}{W^p})^{\alpha}(\frac{1}{P_I})^{1-\alpha}\Big]^{1-\theta} \tag{31}$$

Hence, employment can be expressed as follows:

$$\begin{aligned} L &= \frac{1}{W^p}\Big(\frac{M}{\chi} - Sy_T\Big) \\ &= \frac{1}{2\chi}\Bigg(\chi y_T H(P_I) + \sqrt{\big(\chi y_T H(P_I)\big)^2 + 4\frac{\chi^2(\chi+1)}{\eta}}\Bigg) - y_T\frac{S}{W^p} \end{aligned} \tag{52}$$

Therefore, from (41), (42), (32), (43), I have shown that at optimal point, welfare is independent from the predetermined wage level and labor market's friction conditions but still affected by P_I.

Figures 4 and 5 show the responses of money supply, the nominal exchange rate, CPI, composite consumption, labor, and welfare of households when there are fluctuations in P_I with $\theta = 1.5$ and $\theta = .5$, respectively. Like the flexible wage environment, when P_I rises, labor decreases (increases) with $\theta > 1$ ($\theta < 1$).

Figure 6 compares households' welfare between two values of θ: 1.5 and 0.5. Like the case in flexible wage setting, welfare under $\theta = 1.5$ is higher.

Figures 7 and 8 compares households' welfare between flexible wage and predetermined wage under the same value of θ for two cases (1.5 and 0.5). It is shown that welfare under the predetermined wage setting case strictly dominates the flexible wage setting case. In other words, predetermined wage provides the Ramsey monetary authority effective tools to improve welfare upon flexible wage allocations.

4.1 A particular case

In this subsection, I consider a particular case where labor supply is very elastic, that is the dis-utility of labor is linear [7]. Formally, the utility of a representative household can be expressed as:

$$U(i) \equiv ln(C(i)) + \chi ln\Big(\frac{M(i)}{P}\Big) - \eta L(i) \tag{53}$$

In this case, by the same token, the solution for optimal money supply and derived nominal exchange rate are obtained as follows:

$$M = \frac{(1+\chi)\chi}{\eta} W^p \tag{54}$$

$$\frac{1}{S}\Big[1 - \frac{\alpha\gamma}{\gamma + (1-\gamma)G(S)}\Big] = \frac{\eta y_T}{(1+\chi)W^p} \tag{55}$$

It is straightforward that except the implication that the optimal money supply is independent from P_I, other implications of the previous section on the predetermined wage setting hold.

Corollary 4.1: *Under the assumptions that labor dis-utility is linear and wages are predeterminedly set wage, the Ramsey optimal money supply is independent from terms of trade fluctuations.*

To better understand the role of money supply, I rewrite the Ramsey optimal money supply as:

$$M = \frac{(1+\chi)\chi}{\eta} W^p = \big[(1+\chi)\frac{\phi}{\phi - 1}\big]\big[\frac{\chi}{\eta}\frac{\phi - 1}{\phi} W^p\big] \tag{56}$$

[7] This kind of utility function can be considered to represent the indivisible labor as Hansen (1985)

The second part on the RHS is the amount of money supply as if it would be conducted under the flexible wage setting environment. the first part, which is greater than one, is called the "multiplier". By utilizing the predetermined wage environment, the Ramsey monetary authority can supply a relatively higher amount of money than he could under flexible wage setting by this multiplier, which helps to overcome not only the distortions caused by market power in the labor market (presented by $\frac{\phi-1}{\phi}$) but also takes advantage of real balances in utility function (presented by parameter χ) to improve household's welfare.

Notice that, since employment is increasing with money supply, employment is also higher compared to flexible wage setting environment that is consistent with traditional Keynesian theories.

5 Concluding remarks

This paper studies the Ramsey type optimal monetary policy with frictions in the labor market in the context of a small open economy in response to terms of trade fluctuations with two types of wage setting. The paper shows that predetermined wage setting environment provides the monetary authority effective tools to improve welfare upon distorted flexible wage setting allocations, which is consistent with traditional Keynesian point of view. However, the dimension of monetary policy operations is crucially depends on the value of the elasticity of substitution between tradable and nontradable goods.

This paper offers some possible extensions. First, since the model is static, ones can extend it to a dynamic system to enrich the analysis. Second, the model just considers two extreme cases: flexibly wage setting and predetermined wage setting, therefore, introducing heterogeneous in wage setting, i.e., a fraction of households follows flexible the former setting while others have sticky wage may provide further insights of monetary policy in the small open economy.

Third, one can also extend the model to the context of emerging developing economies by incorporating of incomplete financial markets, financial constraints, and dollarization for further policy analysis.

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Appendix

Price of nontradable goods

Under competition market with free entry, price of nontradable goods is equal to the (minimum) cost of producing one unit of nontradable goods.

$$min \quad WL + SP_I I \tag{57}$$

subject to

$$AL^{\alpha} I^{1-\alpha} = 1$$

F.O.Cs implies:

$$\frac{W}{SP_I} = \frac{\alpha}{1-\alpha}\frac{I}{L}$$

Substitute this into production function, we obtain the demand for labor and intermediate goods as:

$$L = \frac{Y_N}{A}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\left(\frac{SP_I}{W}\right)^{1-\alpha} \quad I = \frac{Y_N}{A}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha}\left(\frac{SP_I}{W}\right)^{-\alpha} \tag{58}$$

Therefore, the price of nontradable goods is:

$$P_N = \kappa\frac{W^{\alpha}(SP_I)^{1-\alpha}}{A} \tag{59}$$

where $\kappa \equiv (\frac{1}{1-\alpha})^{1-\alpha}(\frac{1}{\alpha})^{\alpha}$

Implicit labor demand from nontradable sector

Under competitive market assumption, firms in nontradable goods sector take P_N as given and choose $L(i)$ to maximize their profits, that is:

$$max \quad \Pi = P_N Y_N - \int_0^1 W(i)L(i)di - SP_I I$$

subject to:

$$Y_N = AL^{\alpha}I^{1-\alpha}L = \Big(\int_0^1 L_t(i)^{\frac{\phi-1}{\phi}}dj\Big)^{\frac{\phi}{\phi-1}} \tag{60}$$

F.O.Cs for $L(i)$ imply:

$$\frac{\partial \Pi}{\partial L(i)} = P_N\frac{\partial Y_N}{\partial L}\frac{\partial L}{\partial L(i)} - W(i) = 0 \tag{61}$$

Substitute $\frac{\partial Y_N}{\partial L}\frac{\partial L}{\partial L(i)}$, we obtain the implicit labor demand as:

$$W(i) = \alpha\frac{Y_N}{L}\Big(\frac{L(i)}{L}\Big)^{\frac{-1}{\phi}}P_N \tag{62}$$

Choice of C_N, C_T for given C

For a given composite consumption C, households choose C_N, C_T so that they minimize the expenditure with given P_T, P_N, that is

$$min \quad P_TC_T + P_NC_N$$

subject to:

$$C \equiv \left[(1-\gamma)^{\frac{1}{\theta}}(C_{T,t})^{\frac{\theta-1}{\theta}} + (\gamma)^{\frac{1}{\theta}}(C_{N,t})^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{63}$$

F.O.Cs implies:

$$\frac{C_N}{C_T} = \frac{1-\gamma}{\gamma}\Big(\frac{P_T}{P_N}\Big)^{\theta}$$

Substitute this condition back to composite consumption function, we obtain the choice for $C_T(i), C_N(i)$.

Relations between M and S, M and L

We can rewrite (29) as:

$$\left[1 - \frac{\alpha}{1 + \frac{(1-\gamma)}{\gamma}\Big[\frac{A}{\kappa}(\frac{1}{W^p})^{\alpha}(\frac{1}{P_I})^{1-\alpha}\Big]^{1-\theta}S^{\alpha(1-\theta)}}\right]\frac{M}{\chi} = Sy_T \tag{64}$$

Let denote

$$B \equiv \frac{(1-\gamma)}{\gamma}\Big[\frac{A}{\kappa}(\frac{1}{W^p})^{\alpha}(\frac{1}{P_I})^{1-\alpha}\Big]^{1-\theta}.$$

then we can write above equation as:

$$M = \chi y_T \frac{S(1+BS^{\alpha(1-\theta)})}{1-\alpha+BS^{\alpha(1-\theta)}} \tag{65}$$

Therefore,

$$\frac{dM}{dS} = \frac{\chi y_T\Big((1-\alpha)+(BS^{\alpha(1-\theta)})^2+\big[1+(1-\alpha)-\alpha^2(1-\theta)\big]BS^{\alpha(1-\theta)}\Big)}{(1-\alpha+BS^{\alpha(1-\theta)})^2} \tag{66}$$

Since $\alpha < 1, \theta < 1$, $\frac{dM}{dS} > 0$, which implies $\frac{dS}{dM} > 0$

Employment can be expressed as:

$$L = \frac{1}{\chi W^p}\Big(M - \chi y_T S\Big) \tag{67}$$

Hence,

$$\frac{dL}{dM} = \frac{1}{\chi W^p}\Big(1 - \chi y_T\frac{dS}{dM}\Big) = \frac{1}{\chi W^p}\frac{dS}{dM}\Big(\frac{dM}{dS} - \chi y_T\Big) \tag{68}$$

Substitute $\frac{dM}{dS}$ from above to this equation, we obtain:

$$\frac{dL}{dM} = \frac{y_T}{W^p}\frac{dS}{dM}\left(\frac{(1-\alpha)-(1-\alpha)^2+\alpha\big[1-\alpha(1-\theta)\big]BS^{\alpha(1-\theta)}}{(1-\alpha+BS^{\alpha(1-\theta)})^2}\right) \tag{69}$$

which is positive since $\alpha < 1, \theta < 1$, $\frac{dS}{dM} > 0$.

Solving for optimal money supply and S

Setting the Lagrangian function for this optimal problem, λ is Lagrangian multiplier:

$$\mathcal{L} = ln(M) - ln(S) - \frac{1}{1-\theta}ln\Big((1-\gamma)+\frac{\gamma}{G(S)}\Big) - \frac{\eta}{2(1+\chi){W_p}^2}\Big(\frac{M}{\chi} - Sy_T\Big)^2$$
$$+\lambda\Big\{\chi S y_T - M\Big[1 - \frac{\alpha\gamma}{\gamma+(1-\gamma)G(S)}\Big]\Big\} \tag{70}$$

Applying Kuhn-Tucker conditions, we obtain FOCs:

$$\frac{1}{M} - \frac{\eta}{\chi(\chi+1)W_p^2}(\frac{M}{\chi} - Sy_T) - \lambda\Big(1 - \frac{\alpha\gamma}{\gamma + (1-\gamma)G(S)}\Big) = 0 \tag{71}$$

$$-\frac{1}{S} + \frac{1}{1-\theta}\frac{\gamma}{1-\gamma+\frac{\gamma}{G(S)}}\frac{G'(S)}{G(S)^2} + \frac{\eta}{(1+\chi)W_p^2}y_T(\frac{M}{\chi} - Sy_T)$$
$$+\lambda\left(\chi y_T - M\frac{\alpha\gamma(1-\gamma)G'(S)}{[\gamma+(1-\gamma)G(S)]^2}\right) = 0 \tag{72}$$

$$M\Big(1 - \frac{\alpha\gamma}{\gamma + (1-\gamma)G(S)}\Big) = \chi S y_T \tag{73}$$

where

$$G'(S) \equiv \frac{\partial G(S)}{\partial S} = \alpha(1-\theta)\frac{G(S)}{S} \tag{74}$$

Using definition of $G'(S)$, we can rewrite the condition (55) as:

$$-\frac{1}{S}\Big[1 - \frac{\alpha\gamma}{\gamma+(1-\gamma)G(S)}\Big] + \frac{\eta}{(1+\chi)W_p^2}y_T(\frac{M}{\chi} - Sy_T)$$
$$+\lambda\left(\chi y_T - M\frac{\alpha\gamma(1-\gamma)G'(S)}{[\gamma+(1-\gamma)G(S)]^2}\right) = 0 \tag{75}$$

Consider the case when $\lambda = 0$, then (54) and (58) give the following solutions for M, S:

$$\frac{1}{M} = \frac{\eta}{\chi(\chi+1)W_p^2}(\frac{M}{\chi} - Sy_T) \tag{76}$$

$$\frac{1}{S}\Big[1 - \frac{\alpha\gamma}{\gamma+(1-\gamma)G(S)}\Big] = \frac{\eta y_T}{(1+\chi)W_p^2}(\frac{M}{\chi} - Sy_T) \tag{77}$$

These M, S satisfy the condition (56), hence, they determine the solution M, S of the maximization problem.

Consider the case where $\lambda > 0$, then substitute (37) into (55), we obtain:

$$-\frac{\chi y_T}{M} + \frac{\eta y_T}{(1+\chi)W_p^2}(\frac{M}{\chi} - Sy_T) + \lambda\Big(\chi y_T - M\frac{\alpha\gamma(1-\gamma)G'(S)}{[\gamma+(1-\gamma)G(S)]^2}\Big) = 0$$

$$-\chi y_T\Big(\frac{1}{M} - \frac{\eta}{\chi(\chi+1)W_p^2}(\frac{M}{\chi} - Sy_T)\Big) + \lambda\Big(\chi y_T - M\frac{\alpha\gamma(1-\gamma)G'(S)}{[\gamma+(1-\gamma)G(S)]^2}\Big) = 0 \tag{78}$$

Substitute (54) into this equation, we have:

$$\lambda\frac{\alpha\gamma}{\gamma+(1-\gamma)G(S)}\Big[\chi y_T - M\frac{(1-\gamma)G'(S)}{\gamma+(1-\gamma)G(S)}\Big]=0 \tag{79}$$

Notice that we are considering the case $\lambda > 0$, and substitute $G'(S)$ from (38) into this equation, we obtain:

$$\chi y_T = \frac{M}{S}\frac{\alpha(1-\theta)(1-\gamma)G(S)}{\gamma+(1-\gamma)G(S)} \tag{80}$$

Substitute ratio M/S from (37), we obtain equation for G(S) as:

$$G(S) = \frac{\gamma(1-\alpha)}{(1-\gamma)\big[\alpha(1-\theta)-1\big]} \tag{81}$$

Since, $\theta < 1, \alpha < 1, \gamma < 1$, the LHS of above equation is negative while $G(S)$ is always positive, hence, there is no solutions when $\lambda > 0$.

Simulation

To see how CPI, nominal exchange rate S, optimal money supply, and household's welfare react when there are fluctuations of intermediate goods price P_I, I do some simulations. First, let P_I take on values belonging to $\{0.1; 0.2; 0.3; ...; 2.9; 3\}$ with uniform distribution.

Second, for the parameters to be calibrated, I set $\alpha = 0.6$ so that the share of intermediate goods in production is 40 percent, which is consistent with the estimates for intermediate imports of many literature in Asian. I also set $\gamma = 0.5$ for the reasons that nontradable sector are half of GDP.

Third, I consider three cases for the values of elasticities of substitution between tradable and nontradable goods relative to unity, they are 0.5, 1, and 1.5.

Fourth, I assume that the coefficients of real balances χ, labor dis-utility η in utility function, and productivity in Cobb-Douglas function are equal to unity. I pick value of 5 for the elasticity of substitution between labor varieties.

Finally, to calculate solution S from (24), (38), I use Broyden's method, whose Matlab codes is provided by CompEcon Toolbox in Miranda and Fackler (2002).

Figure 1: Responses to P_I when theta =1.5

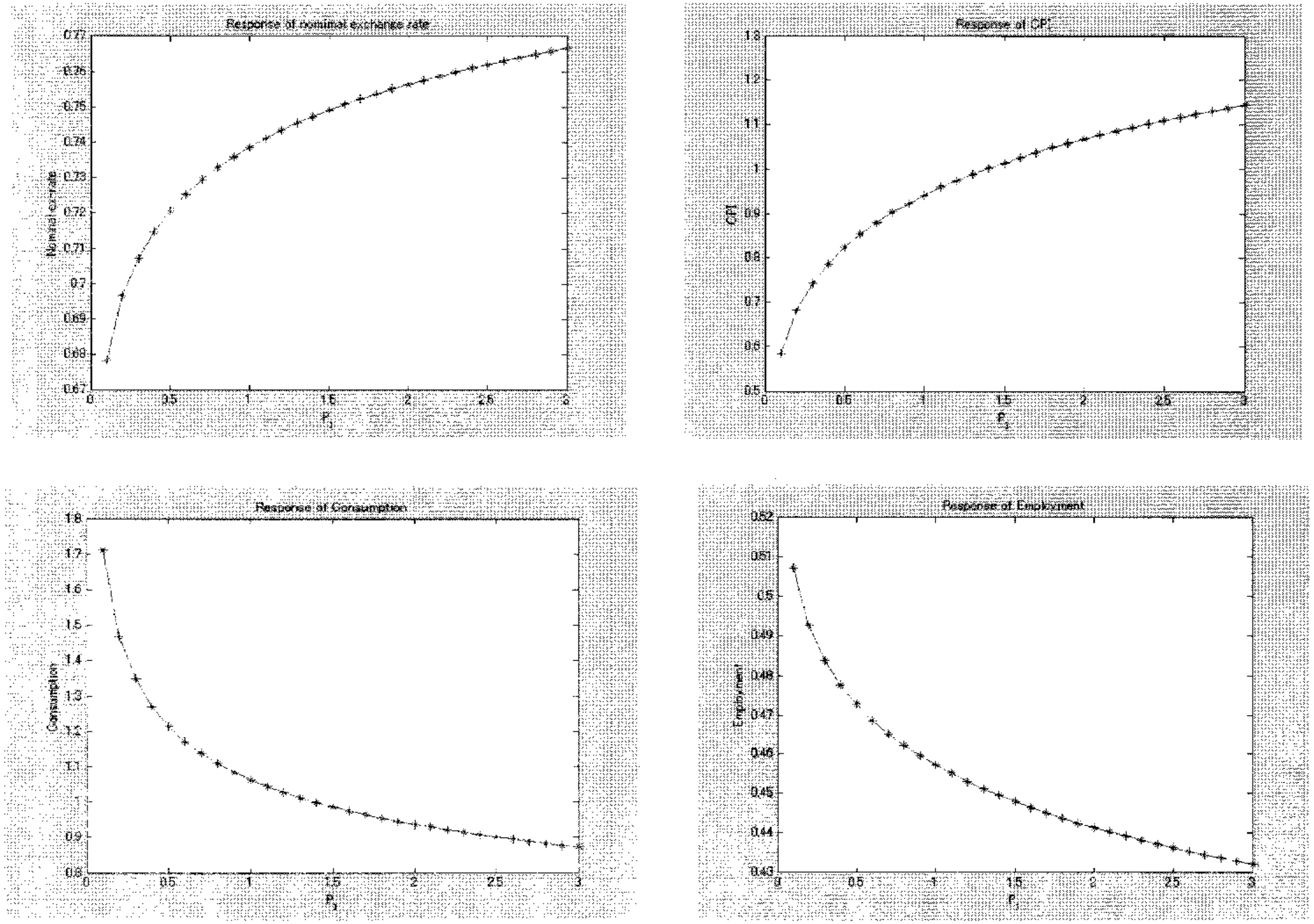


Figure 2: Responses to P_I when theta =0.5

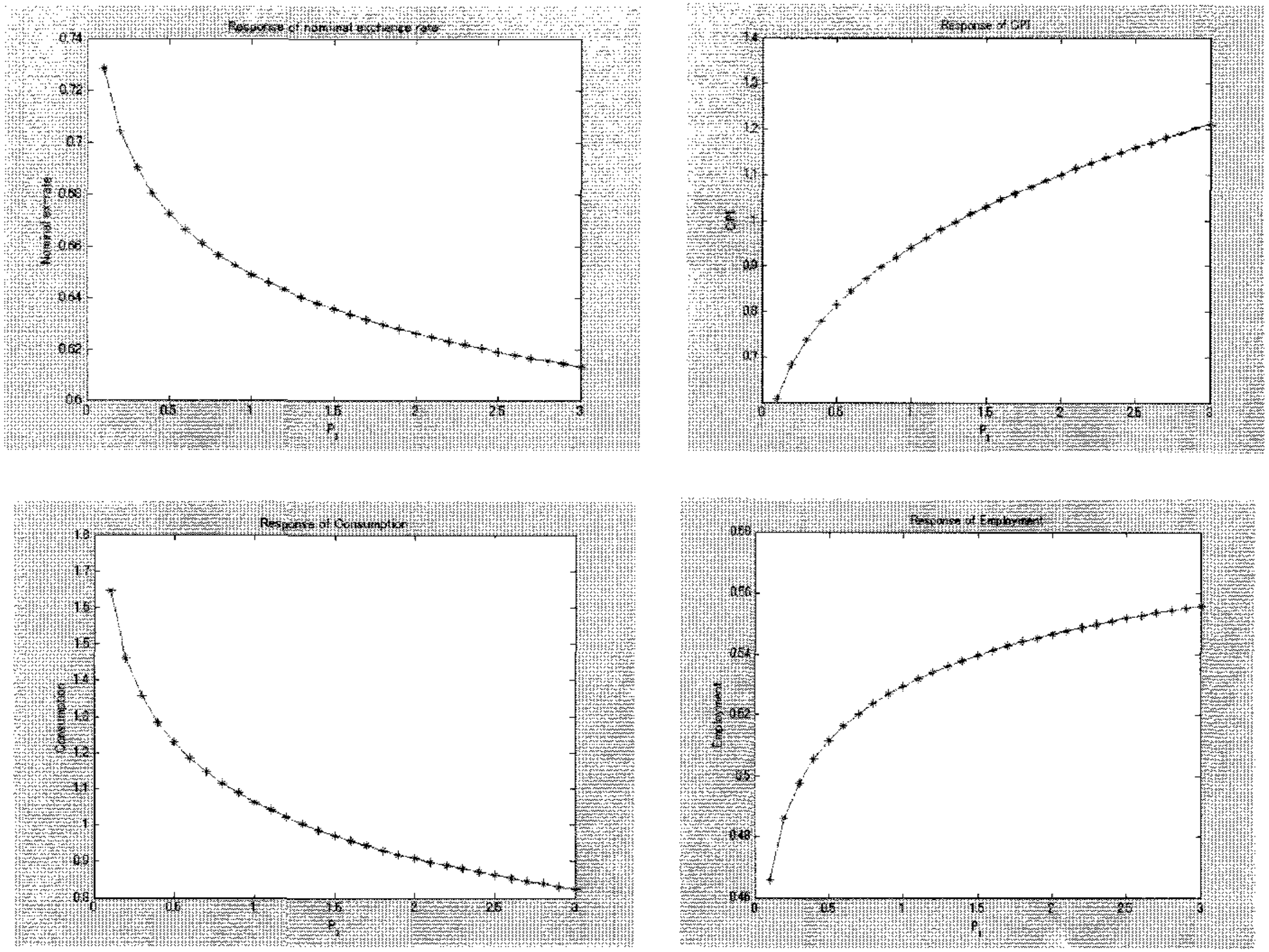


Figure 3: Responses of Welfare

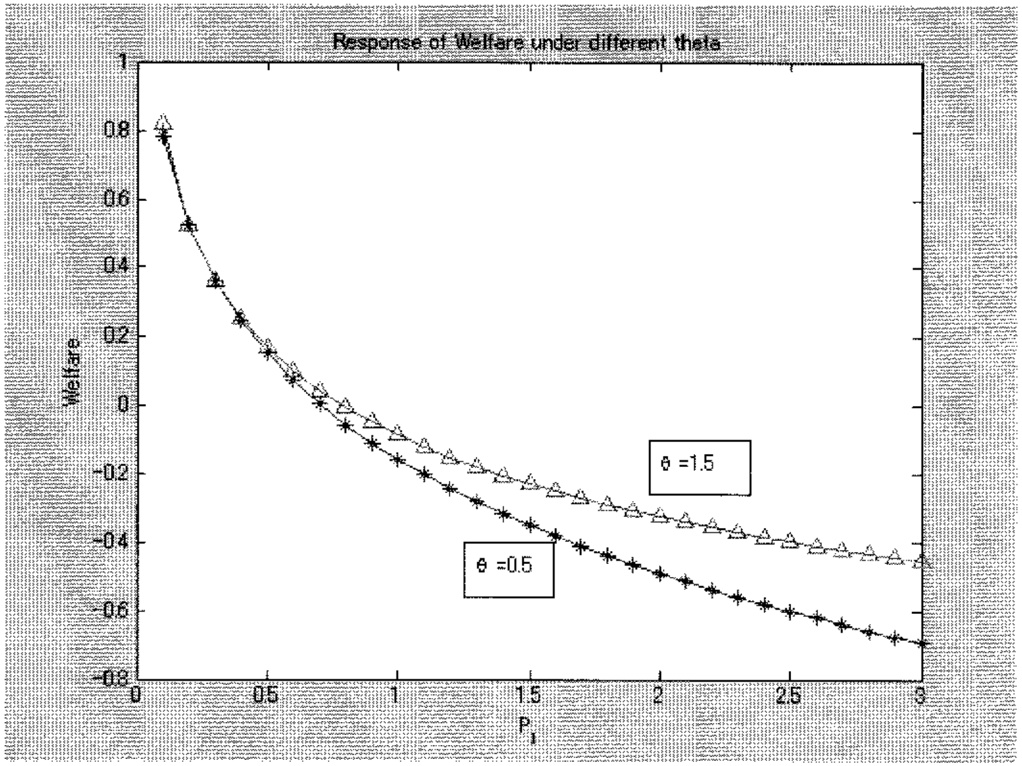
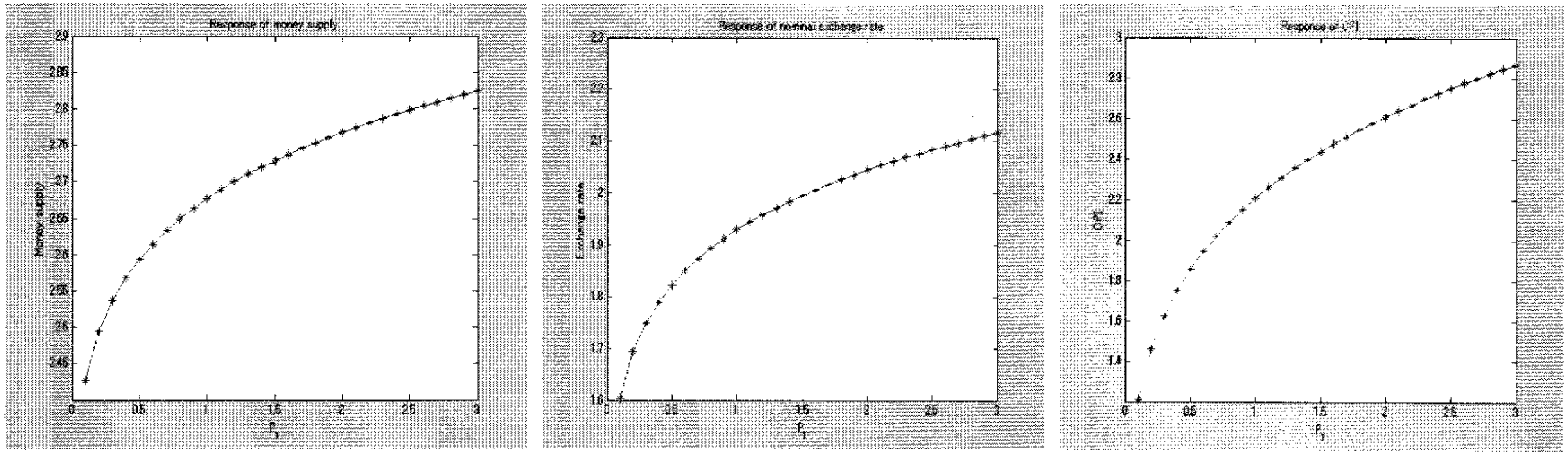


Figure 4: Responses under predetermined wage with theta =1.5



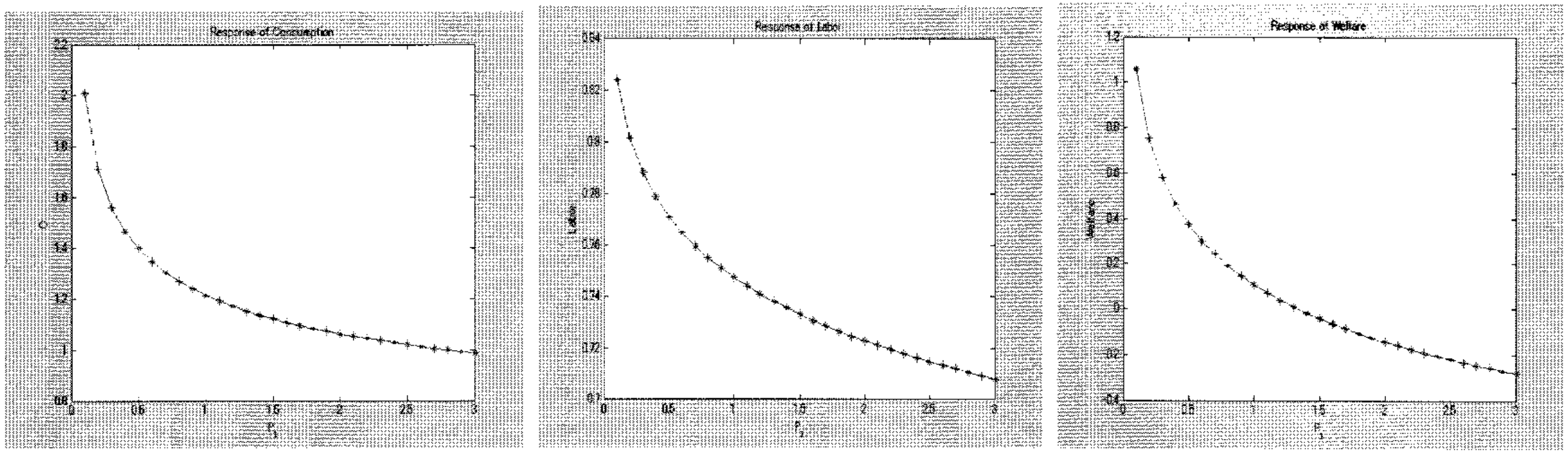


Figure 5: Responses under predetermined wage with theta =0.5

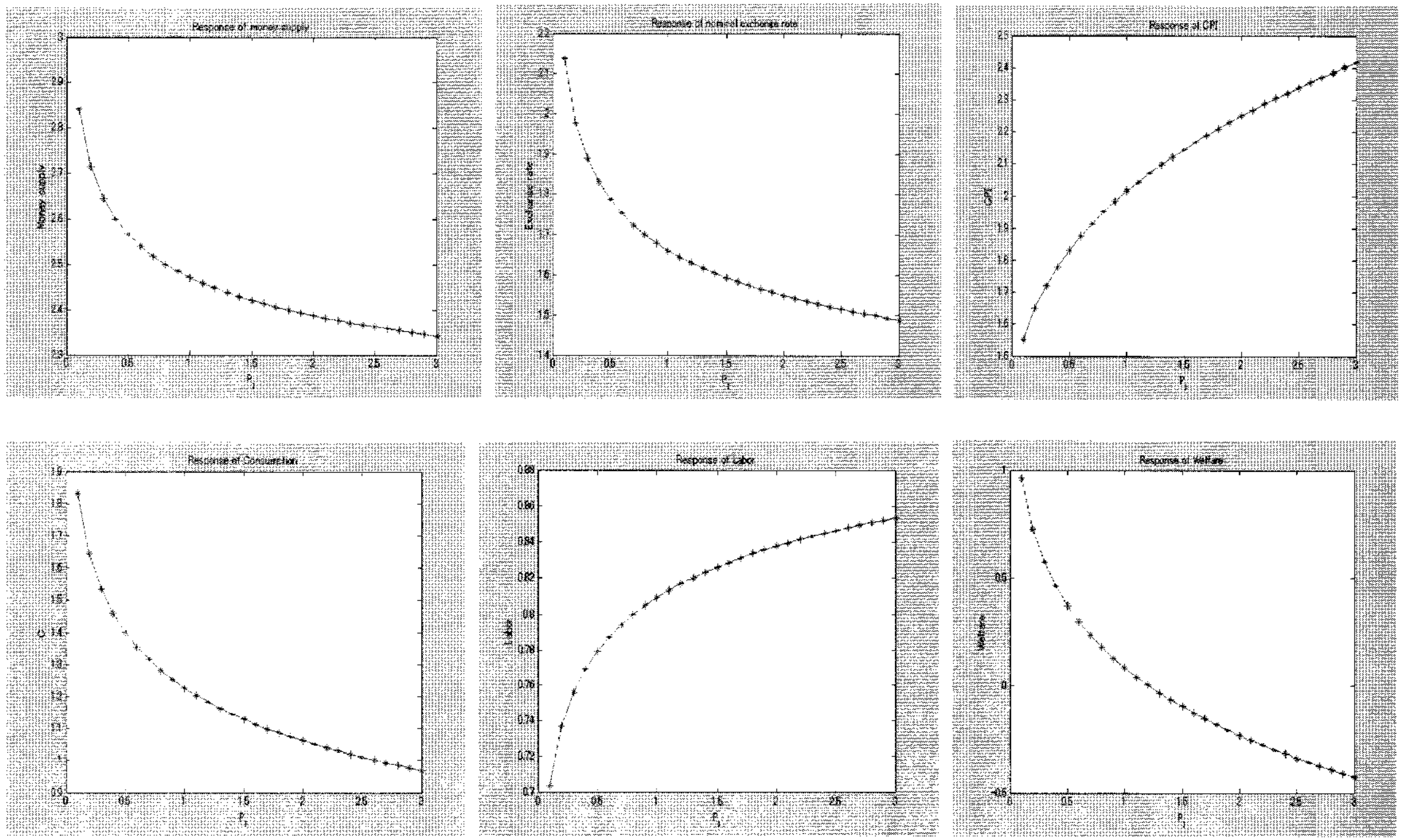


Figure 6: Responses of Welfare under predetermined wage

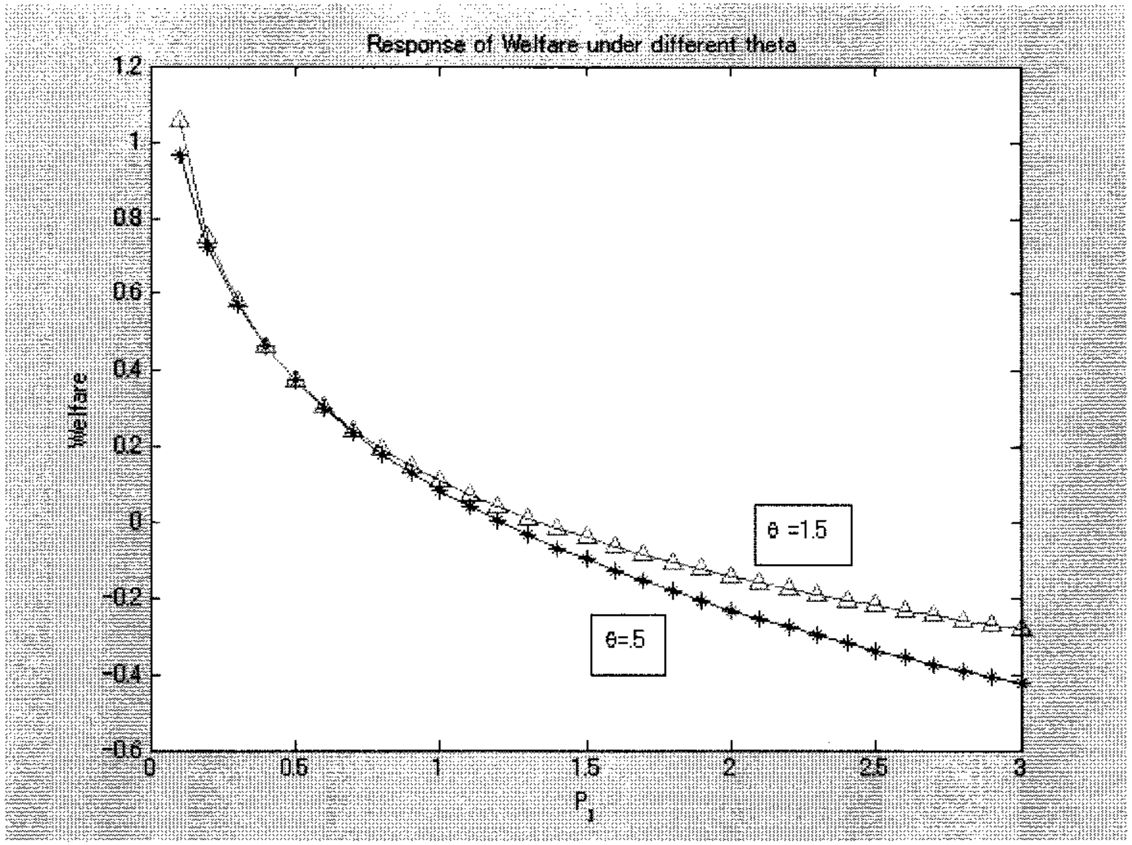


Figure 7: Comparison of welfare between flexible and predetermined wage

Theta = 1.5

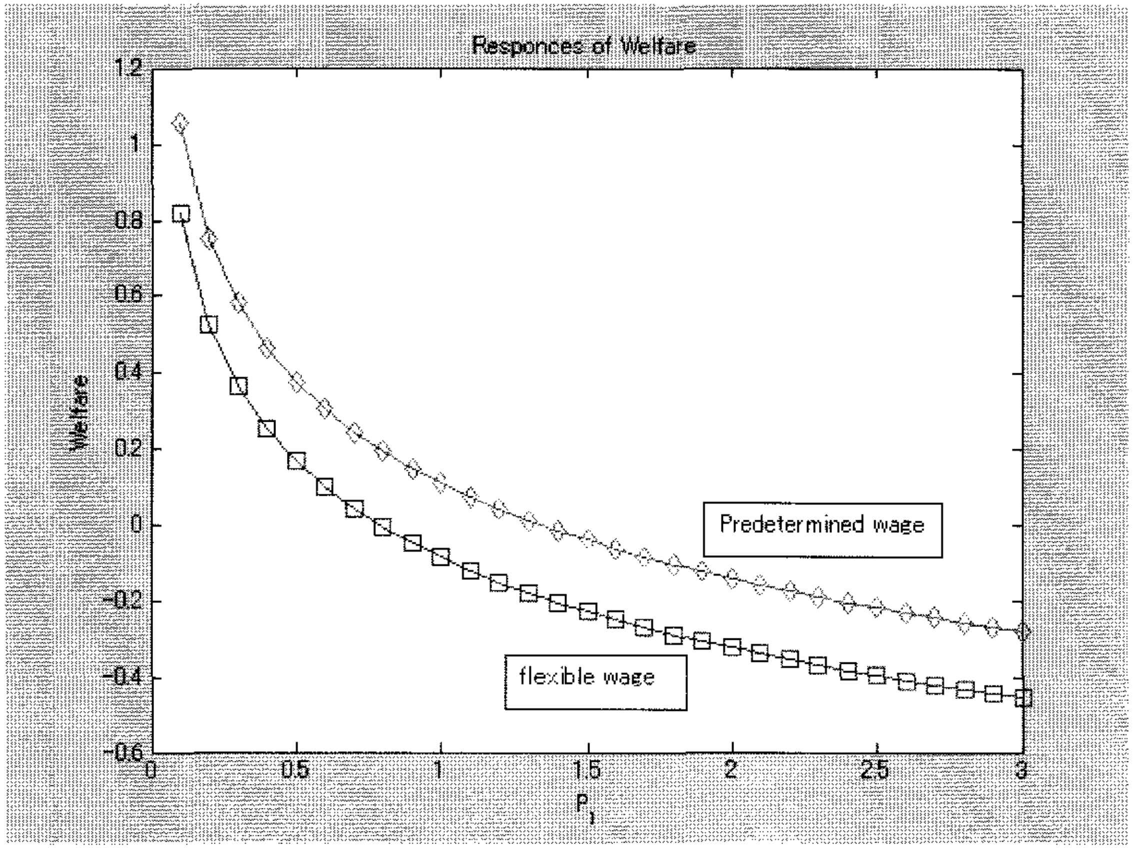
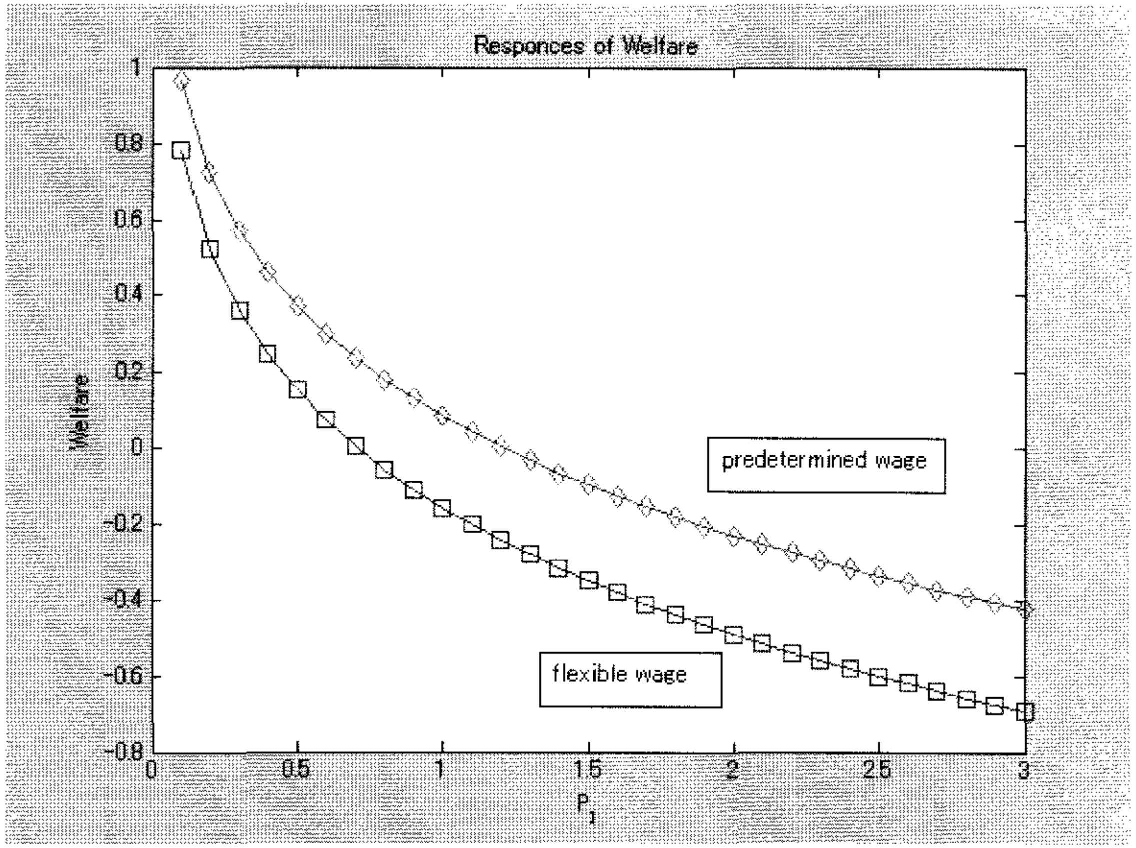


Figure 8: Comparison of welfare between flexible and predetermined wage

Theta = 0.5



Responces of Welfare
Welfare
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
0.5
1
1.5
2
2.5
3
P_1
predetermined wage
flexible wage