1 Introduction

Current economic and financial turbulence that was originally triggered and then exacerbated by problems of the housing market has led to renewed attention to the relations between housing and the macroeconomic business cycle. In fact, variance in the housing sector always accounts for a significant share of variance in output in many economies, [1] and in the past 60 years, eight out of ten recessions in the US were preceded by substantial problems in housing, therefore, in a well-circulated paper, Leamer (2007) simply puts it: "Housing *IS* the business cycle".

Nonetheless, as argued by Iacoviello (2010) and several others, academic work addressing the housing market and the macroeconomic business cycle did not receive quite satisfactory attention before the onset of the global financial crisis 2007-8. Business cycle models, started with the seminal work of Kydland and Prescott (1982), often combine household investment[2] that include residential or housing investment and other non-housing durable consumption with business investment all together into one kind of aggregate investment. Though these types of standard one-sector real business cycle models obtained success in explaining certain aspects of business cycles, [3] there are good reasons for the first wave of the literature to distinguish between household investment and business investment in addressing business cycles. First, the stock of household capital or home capital is higher than the stock of business capital. Second, typical households often spend almost as much time on unpaid work at home such as cooking, cleaning, and parenting as on work for pay in the labor market. Consequently, home production using home capital and its devoted home work have been estimated to be between

[1]Housing demand shocks account for 20-25% of variance in GDP in the U.S and Japan (IMF, 2008)

[2]This term is first used by Greenwood and Hercowitz (1991).

[3]See Cooley and Prescott (1995) for further discussion.

20% to 50% of GDP and understanding the dynamics of *home economics* is very important for business cycle studies.

The literature that incorporates home production, however, often have difficulty in explaining the co-movements between dis-aggregated investment because there is a strong incentive to switch labor and production between sectors in response to sector-specific productivity shocks. Therefore, in order to solve the co-movement problem, Greenwood and Hercowitz (1991) and Baxter (1996) assume reversibility between residential and business capital and also highly correlated productivity shocks between two sectors. Fisher (1997) assumes complementarity between the household and business capital in goods production, specifically, a nonlinear function for transforming output into non-durable consumption goods, new consumer durables, and new physical capital. Chang (2000) argues that if there are adjustment costs in capital accumulation and substitutability between leisure time and durable goods in home production, then when households work more in periods of high productivity they also demand more durables.[4] These authors, however, often have difficulty in accounting for the relatively high volatility of home investment that includes housing investment.

Moreover, Davis and Heathcote (2005; henceforth DH) argues that home production models are not quite fit to examine the dynamics of the housing market because of several reasons. First, housing and other consumer durables are produced using different technologies and housing production are more labor intensive. Second, housing is much more durable than consumer durables. [5]Third, housing investment is about two times more volatile than that of consumer durables. Fourth, housing prices are about 4 times more volatile than the price of consumer durables; housing prices are positively correlated with GDP while durable goods prices are negatively corre-

[4]See Charles Leung (2004) for further literature review and explanations.

[5]The depreciation rate for housing is 1.6% per year vs. 21.4% for consumer durables.

lated with GDP.[6]

To explain both the co-movement of disaggregated investment and and the high volatility of housing investment, DH construct and simulate a pretty standard neoclassical growth model where housing and the other sectors all use three intermediate goods but in different proportions. The high volatility is the result of the calibration that the housing construction sector uses a relatively higher proportion of intermediate goods which are more volatile. In particular, since housing stock is highly durable, households are able to concentrate housing investment in the periods of high productivity and also because housing production is labor intensive it is possible to expand housing production quickly. The co-movement of housing investment and business investment of DH is, however, attributed by the introduction of land, which is in fixed supply hence acts as an adjustment cost because it becomes relatively expensive during construction booms. DH, however, fails to explain a positive correlation between housing prices and housing investment, a well-documented regularity. The main reason is that the DH model implies that households tend to buy more houses as they become relatively cheaper in economic booms. This implication is not supported by empirical evidence.

This paper explores the *open economy* environment in explaining the co-movement of disaggregated investment and the positive correlation between housing prices and housing investment and to our knowledge, it is the first paper that addresses the co-movements in business cycles with housing in an open economy. The motivation is straightforward. Since in a closed economy model where the representative household has no access to the international financial market, so with limited resources it has to choose to invest in the sector that is more productive and therefore withholding the investment in

[6] In addition, in an interesting paper, Barsky et al. (2007) demonstrates that the behaviors of New Keynesian models depend crucially on the stickiness of long-lived housing prices no matter how small this sector is.

the other sector with lower productivity. In other words, in a closed economy model there is a strong incentive to switch labor and production between sectors in response to sector-specific productivity shocks, hence causing negative co-movements. That explains why in the DH closed economy models, in addition to positively correlated TFP shocks of the intermediate sectors, land that acts as an adjustment cost increases the cost of shifting between sector, helping produce positive co-movements. By contrast, in our model, openness enables the representative household to have access to the international financial market, hence allowing it to borrow, if necessary, to invest in both sectors when positive shocks affects both sectors even with different magnitudes. For example, when a positive productivity shock hits the non-housing sector, causing a gap in the productivity between the two sectors, consequently leading on impact to an increase in the non-housing good production, a decrease in housing production, and an increase in the real housing price. However, since there are positive productivity spillovers and also because the representative household is able to borrow from the intentional financial market to invest in the housing sector's specific non-housing capital, the production of housing will increase from the second period after the shock. As a result, housing investment will positively co-move with the housing price and also with non-housing investment. Meanwhile, since housing in our model is labor intensive, has very low depreciation rate, and is subject to high variance TFP shocks, which are all similar with the DH model, housing investment is highly volatile.

In short, this paper successfully shows that a pretty standard open economy model can generate both a positive co-movement between disaggregated investment and relatively high volatility of housing investment even without the introduction of land. In addition, openness also helps generate a positive correlation between housing investment and house prices, the regularity that the DH model with a closed economy assumption fails to explain.

This paper is organized as follows. Section 2 constructs a standard two-sector open economy model, a model that can be considered as a reduced form of DH model but without land. Section 3 calibrates the model re-using the calibration of DH. Section 4 analyzes and simulates the model dynamics and Section 5 concludes.

2 The Model

To study the role of the open economy assumption, I construct a standard two-sector open economy model where housing is a durable and non-tradable good as whereas the other good is non-durable but tradable.

2.1 Production

At time t, tradable and non-durable goods are produced by a technology that is the Cobb-Douglas function of previously installed sector-specific capital, k^c_{t-1}, and labor, l^c_t, as follows:

$$y_t = A^c_t (k^c_{t-1})^{\alpha_c} (l^c_t)^{1-\alpha_c} \tag{2.1}$$

Output from the tradable and non-durable sector then can be used as non-durable consumption, c_t, and investments on two types of non-housing capitals, i^c_t, i^h_t (for k^c_t, k^h_t, respectively), and can also be exported as, nx_t.

Housing, which is durable but non-tradable, is produced or built using previously installed sector-specific capital, k^h_{t-1} and labor, l^h_t, as follows:

$$b_t = A^h_t (k^h_{t-1})^{\alpha_h} (l^h_t)^{1-\alpha_h} \tag{2.2}$$

where A^c_t, A^h_t denotes the exogenous stochastic productivity shocks for tradable and housing sectors that follows an autoregressive law of motion:

$$\begin{bmatrix} log(A^c_{t+1}) \\ log(A^h_{t+1}) \end{bmatrix} = \begin{bmatrix} a_{cc} & a_{ch} \\ a_{hc} & a_{hh} \end{bmatrix} \begin{bmatrix} log(A^c_t) \\ log(A^h_t) \end{bmatrix} + \begin{bmatrix} \epsilon^c_{t+1} \\ \epsilon^h_{t+1} \end{bmatrix} \tag{2.3}$$

where a_{cc}, a_{hh} measures the persistence in productivity shocks in the sector a, h, and a_{ij} measures the degree of spillover from the sector i to sector j $(i, j \in \{c, h\})$. $\epsilon^c_t, \epsilon^h_t$ are innovations that obeys the following process:

$$\begin{bmatrix} \epsilon^c_t \\ \epsilon^h_t \end{bmatrix} \sim N(0, V) \tag{2.4}$$

where V is the variance matrix of innovations.

Note that in the production of housing (2.2), we dont introduce land, which acts as adjustment costs in the production of the residential sector in DH.

Assume that labor is freely mobile between sectors. We normalize the price of the tradable and non durable good after any history to 1 and denote q_t as the relative price of non-tradable and durable housing. Optimality conditions for tradable goods firms imply:

$$w_t = (1-\alpha_c)\frac{y_t}{l^c_t} = (1-\alpha_c)A^c_t\Big(\frac{k^c_{t-1}}{l^c_t}\Big)^{\alpha_c} \tag{2.5}$$

$$r^c_t = \alpha_c\frac{y_t}{k^c_{t-1}} = \alpha_c A^c_t\Big(\frac{k^c_{t-1}}{l^c_t}\Big)^{\alpha_c-1} \tag{2.6}$$

Optimality conditions for the construction sector imply:

$$w_t = q_t(1-\alpha_h)\frac{b_t}{l^h_t} = q_t(1-\alpha_h)A^h_t\Big(\frac{k^h_{t-1}}{l^h_t}\Big)^{\alpha_h} \tag{2.7}$$

$$r^h_t = q_t\alpha_h\frac{b_t}{k^h_{t-1}} = q_t\alpha_h A^h_t\Big(\frac{k^h_{t-1}}{l^h_t}\Big)^{\alpha_h-1} \tag{2.8}$$

where w_t is the real wage rate and r^c_t, r^h_t are rental rates for sector specific capital, k^c_{t-1}, k^h_{t-1}, respectively.

2.2 Household

The representative household maximizes its expected lifetime utility defined over random sequences of non-durable consumption goods (c_t), housing services from the housing stock (h_t), and leisure ($1 - l_t$):

$$U = E_0 \sum_{t=0}^{\infty} \beta^t U\big(c_t, h_t, (1 - l_t)\big) \tag{2.9}$$

Specifically, period utility for the representative household at date t is assumed to be the following Cobb-Douglas form:[7]

$$U(c_t, h_t, (1 - l_t)) = \frac{(c_t^{\mu_c} h_t^{\mu_h} (1 - l_t)^{(1-\mu_c-\mu_h)})^{1-\sigma} - 1}{1 - \sigma} \tag{2.10}$$

where the parameters, μ_c, μ_h determine the shares of non-durable consumption, housing, and leisure in expenditure.

The budget constraint of the representative household at time t is given by:

$$\begin{aligned} c_t + q_t[h_t - (1 - \delta_h)h_{t-1}] + i_t^c + i_t^h + (1 + r_{t-1})d_{t-1} + \frac{\phi_d}{2}(d_t - \bar{d})^2 \\ \leq w_t l_t + r_t^c k_{t-1}^c + r_t^h k_{t-1}^h + d_t \end{aligned} \tag{2.11}$$

$$\begin{aligned} i_t^c &= k_t^c - (1 - \delta_k)k_{t-1}^c + \frac{\phi_k}{2}\frac{(k_t^c - k_{t-1}^c)^2}{k_{t-1}^c} \\ i_t^h &= k_t^h - (1 - \delta_k)k_{t-1}^h + \frac{\phi_k}{2}\frac{(k_t^h - k_{t-1}^h)^2}{k_{t-1}^h} \end{aligned} \tag{2.12}$$

Each period, the household can borrow internationally traded debt, d_t, subject to a adjustment cost, $\frac{\phi_d}{2}(d_t - \bar{d})^2$,[8] at an exogenous real interest rate,

[7] As discussed in Greenwood et al. (1995) and in also in DH, the Cobb-Douglas form of utility function is consistent with evidence of constant shares of working hours and spending in housing.

[8] The introduction of asset adjustment cost is to induce stationary dynamics in a small open frictionless economy. For more details, see Schmitt-Grohe and Uribe (2003)

r_t. It supplies labor, l_t, at the real wage rate, w_t, and lends sector specific capital, k^c_{t-1}, k^h_{t-1}, to capital markets at prices r^c_t, r^h_t. The household then spreads its income on non-durable consumption goods, c_t, debt repayment, $(1 + r_{t-1})d_{t-1}$, investments on two types of non-housing capitals i^c_t, i^h_t, and housing investment, $q_t(h_t - (1 - \delta_h)h_{t-1})$, where δ_h is the depreciation rate of housing stock. Adjustment costs are introduced as in 2.12 to avoid excessive non-housing investment volatility in response to differences in the domestic-foreign interest rates in a small open economy setting.

The first order conditions for the representative household read:

$$U_{ct}[1 + \phi_k(\frac{k^c_t - k^c_{t-1}}{k^c_{t-1}})] = \beta E_t\{U_{ct+1}[1 - \delta_k + r^c_{t+1} + \frac{\phi_k}{2}((\frac{k^c_{t+1}}{k^c_t})^2 - 1)]\} \quad (2.13)$$

$$U_{ct}[1 + \phi_k(\frac{k^h_t - k^h_{t-1}}{k^h_{t-1}})] = \beta E_t\{U_{ct+1}[1 - \delta_k + r^h_{t+1} + \frac{\phi_k}{2}((\frac{k^h_{t+1}}{k^h_t})^2 - 1)]\} \quad (2.14)$$

$$w_t = -\frac{U_{lt}}{U_{ct}} \quad (2.15)$$

$$U_{ct}(1 - \phi_d(d_t - \bar{d})) = \beta E_t\{U_{ct+1}(1 + r_t)\} \quad (2.16)$$

$$q_t U_{ct} = U_{ht} + (1 - \delta_h)\beta E_t\{q_{t+1}U_{ct+1}\} \quad (2.17)$$

The first two equations are standard optimality conditions for capital with adjustment costs while the third one is a standard labor supply equation. The last two equations present distinguishing features of the borrowing constraint model. Equation (2.16) is an Euler equation that states the marginal utility of current non-durable consumption is equated to the marginal gain of shifting one unit of non-durables to the next period. Equation (2.17) is the efficiency condition for the intratemporal choice of durable housing that requires the household to equate the marginal utility of non-durable consumption, weighted by the relative housing prices and adjustment costs, to the marginal utility of housing services. The marginal utility of housing service consists of two components: (i) the direct utility gain of an additional unit of housing; (ii) the expected utility derived from expanding future consumption by means of re-selling the amount of housing invested in the previous period.

After integrating (2.17) forward, we obtain the following demand function for housing:

$$q_t U_{ct} = E_t \Big\{ \sum_{j=0}^{\infty} [(1-\delta_h)\beta]^j U_{ht+j} \Big\} \tag{2.18}$$

The RHS of (2.18) that is the discounted stream of utility from housing services is equated to the weighted marginal utility of non-durable consumption in the LHS. According to Barsky et al. (2007), there are two reasons that keep this value roughly constant against moderate-lived shocks. First, durable housing with low depreciation rates has high stock-flow ratios so even relatively large changes in housing production over a moderate time period have small effects on the total stock, causing only minor changes in the service flows. Second, if δ_h is sufficiently low, the shadow value will be mainly affected by the marginal utilities of service flows in the distant future. Since the effects of the shock are temporary, the future terms in this equation are close to their steady-state values. Thus, even if there were significant changes in the first few terms of the expansion, they would have a small percentage effect on the present value. As a consequence, under the benchmark, demand for durable housing displays an almost infinite elasticity of intertemporal substitution and the demand curve at any given time is very flat.

2.3 Equilibrium

Given the interest rate, r_t, a competitive equilibrium in this economy is characterized by a sequence of allocations $\{c_t, l_t, h_t, d_t, k_t^c, k_t^h, i_t^c, i_t^h, y_t, b_t, l_t^c, l_t^h\}$ and a sequence of prices $\{q_t, w_t, r_t^c, r_t^h\}$ that satisfy the representative household and firms optimality conditions, the budget constraint, production functions, and the following market clearing conditions.

Labor market clearing:

$$l_t = l_t^c + l_t^h \tag{2.19}$$

Non-tradable durable housing market clearing:

$$b_t = h_t - (1 - \delta_h)h_{t-1} \tag{2.20}$$

Tradable non-durable goods market:

$$c_t + i_t^c + i_t^h + (1 + r_{t-1})d_{t-1} + \frac{\phi_d}{2}(d_t - \bar{d})^2 = y_t + d_t \tag{2.21}$$

The trade balance, housing investment, private consumption expenditures, and aggregate output can be expressed as:

$$nx_t = y_t - c_t - i_t^c - i_t^h + \frac{\phi_d}{2}(d_t - \bar{d})^2 \tag{2.22}$$

$$resi_t = q_t b_t \tag{2.23}$$

$$C_t = c_t + q_t b_t \tag{2.24}$$

$$Y_t = y_t + q_t b_t \tag{2.25}$$

2.4 Calibration

The model period is one year and this paper follows closely, albeit under a reduced form, the calibration of DH, which performs a relatively thorough and detailed calibration for a multiple sector neoclassical growth model based on industry level data in the US.

Preference: Following DH and many other papers in the literature, the inverse of elasticity of substitution in consumption, σ, is set to 2. The parameters, μ_c, μ_h are chosen so that households, in steady state, spend 30% of their endowment hours working in the labor market, and so that the ratio of housing investment over GDP is equal to 5%, which is the average level in the last 30 years[9] in the US.

[9]Data is taken until 2007 before the wake of the global crisis.

Discount factor β is set at the value of 0.951, which hence implies an annual level of the real interest rate at 6% as in DH. This value, together with other parameters, implies that in the steady state the level of private debt is 2 times of annual GDP.

Technology: Parameters pertaining to the production side of this paper are calculated from the calibration of DH. In particular, since the production specifications of this paper can be considered as a reduced form of that in DH, there is a map between parameters in the two papers.[10]. As a result, from the Table 3 of DH, we obtain the values for the share of capital in the production of non-durables and housing, α_c, α_h, as 0.25 and 0.2, respectively, which then implies that even under a reduced form the production of housing in this paper is also more labor intensive.

The annual depreciation rate of non-housing capital, δ_k, and the annual depreciation rate of housing, δ_h, are set to 5.57% and 1.41% as in DH. Capital adjustment cost parameter ϕ_k is chosen such that non-housing investment volatility over that of output in this open economy model matches the value of 2.3 as implied by U.S data. Porforlio adjustment cost parameter ϕ_d is set at the value of 0.08 so that the implied standard deviation of the trade balance output ratio is equal to 0.94, the value in the range calculated in open economy literature. [11]

From the Table 4 of DH, we are able to obtain specifications for the productivity shock process as follows:

$$\begin{bmatrix} log(A^c_{t+1}) \\ log(A^h_{t+1}) \end{bmatrix} = \begin{bmatrix} 0.96 & -0.17 \\ 0.11 & 0.64 \end{bmatrix} \begin{bmatrix} log(A^c_t) \\ log(A^h_t) \end{bmatrix} + \begin{bmatrix} \epsilon^c_{t+1} \\ \epsilon^h_{t+1} \end{bmatrix} \tag{2.26}$$

where the covariance matrix of productivity innovations is:

$$V = \begin{bmatrix} 0.0135 & 0.0063 \\ 0.0063 & 0.0194 \end{bmatrix} \tag{2.27}$$

[10]See the Appendix for details

[11]See, for example, Uribe (2012).

Note that the productivity shock process (2.26) implies that there is a *positive* spillover from the productivity shock of the tradable and non-durable sector to the productivity shock of the non-tradable housing sector while there is a *negative* spillover from the productivity shock of the non-tradable housing sector to the productivity shock of the tradable and non durable sector. This will, as discussed in the next section, dictate different dynamics of the model in response to the two types of productivity shocks.

2.5 Quantitative Analysis

The DSGE model is then solved and simulated by the pertubation method,[12] and Figure 1 and Figure 2 present the impulse responses of selected variables in response to a positive productivity shock of the tradable sector and non-tradable housing sector, respectively.

In particular, in response to a positive productivity shock in the tradable but non-durable sector, A^c, labor shifts on impact from the relatively low productivity housing sector to the higher productivity tradable sector, leading to an increase in the production of the tradable good and a decrease in the production of the housing sector (Figure 1). As a result, the real housing price, the wage rate, and capital rental rates increase. Since there are positive spillovers in productivity from the tradable sector to the housing sector, productivity of the latter sector will increase from the next period. In addition and more importantly, under an open economy environment, the representative household is able to temporarily borrow from the international financial market to invest in *both* two types of non housing capital, k_t^c, k_t^h, when their capital returns increase. Meanwhile, because of positive productivity spillovers and non random walk productivity shock, the gap of productivity between the two sector is closed over time and labor gradually

[12] For details, see Schimitt-Grohe and Uribe (2004).

shifts back to the housing production. Consequently, production of the housing sector and hence housing investment will increase from the next period, leading to positive co-movements between the housing price, housing investment and also non-housing investment in response to a productivity shock in the tradable sector.

By contrast, when there is a positive productivity shock in the non-tradable housing sector, which is relatively labor intensive, labor and production of this sector increase substantially (Figure 2), causing a decline in the relative housing price. Moreover, because there are negative spillovers from the productivity of the housing sector to the productivity of the tradable consumption sector, productivity of the tradable sector and its specific capital returns will decrease from the next period. Consequently, it will encourage the representative household to save by temporarily lending to the international financial market. Also, because housing is non-tradable and relatively labor intensive, has a very low depreciation rate, and its TFP productivity shock is volatile but less persistent. i.e., it dies out quickly as in (2.26), production of housing is also relatively responsive, contributing to a relatively high volatility of housing investment.

Finally, the Table 1 presents the second moments of selected variables implied by the model in comparison with data and those implied by DH. We find that the quantitative results of this pretty standard two sector open economy model are closely similar with those from DH and therefore can account for many well-documented features of the business cycle with a housing sector. In particular, the model generates both positive correlations between housing investment and non-housing investment and between housing investment and output and at the same time a relatively high volatility of housing investment even without the introduction of land. As discussed and quantitatively shown in DH, land, which acts like an adjustment cost, plays a determining role in generating a co-movement between housing investment and

Figure 1: IRs to a Productivity Shock of the Tradable Sector

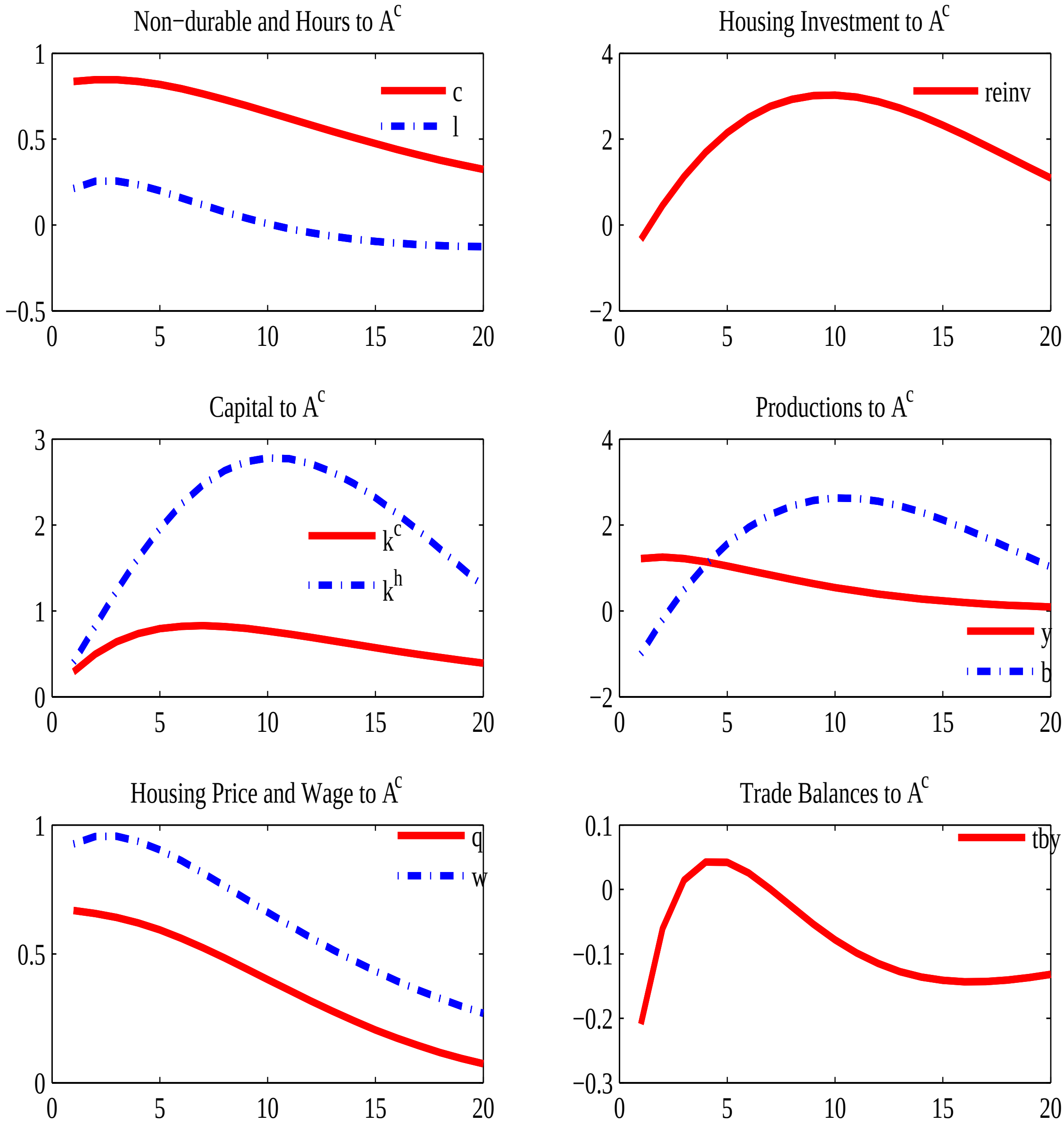
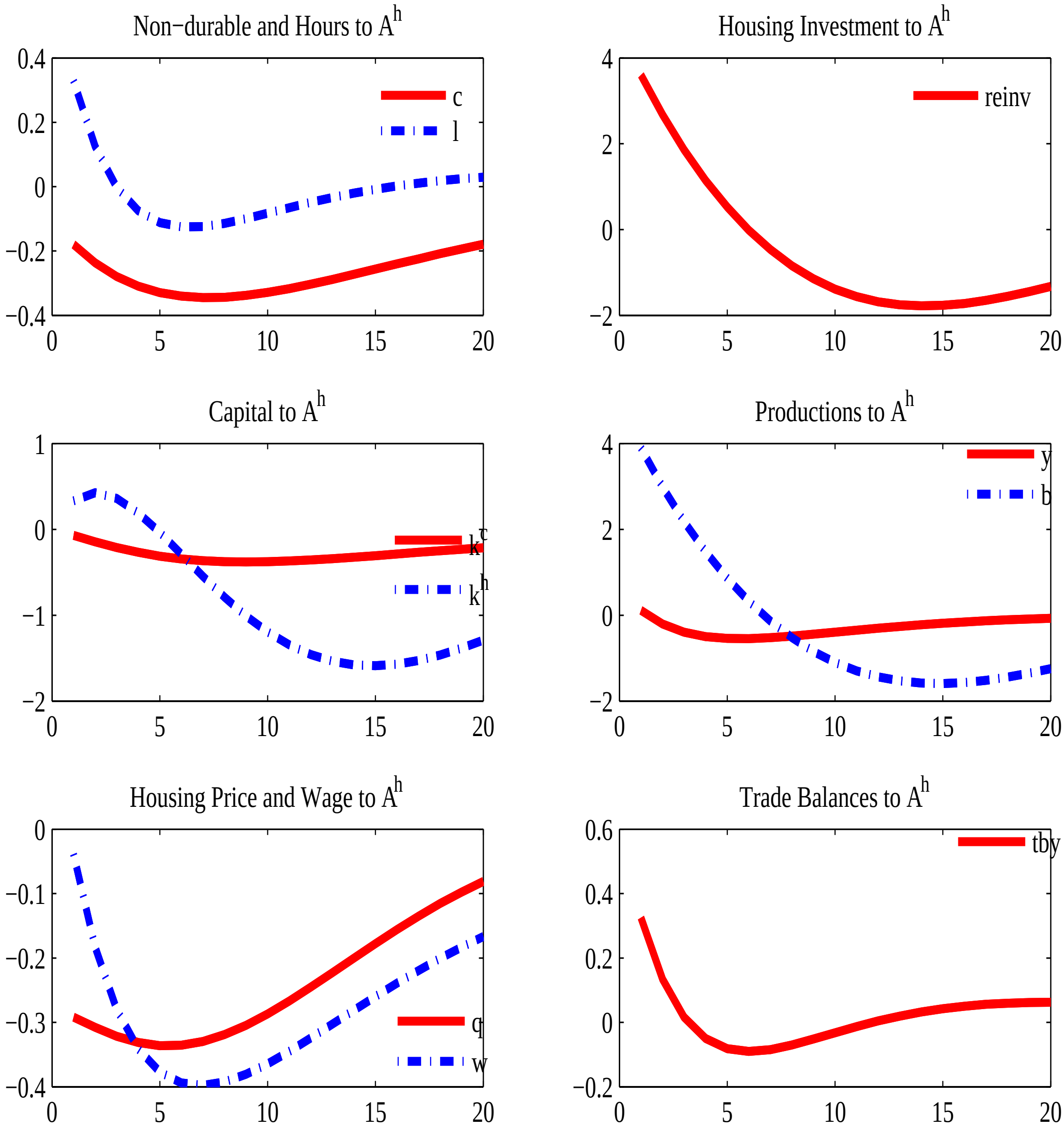


Figure 2: IRs to a Productivity Shock of the Housing Sector



non-housing investment. Therefore, our model quantitatively shows that the open economy assumption that allows an access to the international financial market also is also able to help generate the co-movements even without the introduction of an adjustment cost in producing houses. Meanwhile, since housing is non-tradable, labor intensive, has very low depreciation rate, and is subject to higher variance of productivity shocks, housing investment is relatively more volatile than non-housing investment as in DH.

Notably, we find that the open economy assumption also helps produce a positive correlation between housing prices and housing investment, a well-documented regularity that DH fails to predict. The main reason is that openness enables the representative household to have access to the international financial market, hence allowing it to borrow, if necessary, to invest in both sectors when there is a positive shock hitting both sectors. By contrast, in a closed economy model where the representative household has no access to the international financial market, it has to choose to invest in the sector that is more productive and therefore withholding the investment in the other sector with lower productivity, causing to a negative co-movement. For example, when a positive productivity shock hits the tradable sector, causing a gap in the productivity between the two sector, consequently leading to an increase in tradable good production, a decrease in housing production, and an increase in the real housing price. Because there are positive productivity spillovers and also the representative household is able to borrow from the intentional financial market to invest in the housing sector's specific non-housing capital, the production of housing will increase from the second period after the shock. As a result, housing investment will positively co-move with the housing price.

Table 1: **Business Cycle Properties**

	Data	DH	Model
GDP (%SD)	2.26	1.73	4.38
SD relative to GDP			
C	0.78	0.48	0.90
l	1.1	0.41	0.34
nres	2.3	3.21	2.3
res	5.04	6.12	4.1
tb/y	0.94	-	0.94
hp	1.37	0.4	0.5
Correlations			
nres, res	0.25	0.15	0.34
hp,res	0.34	-0.20	0.49
hp, GDP	0.65	0.65	0.88
C, GDP	0.80	0.95	0.91
C, res	0.66	0.26	0.79
C, nres	0.61	0.91	0.62

Notes: Data and DH are obtained from DH. SD is standard deviation. C: private consumption expenditures, l: labor or hours, nres: non-housing investment, tb: trade balance, tb/y: trade-balance output ratio, res: housing investment, hp: real housing prices. All numbers are in percentage, which is the standard deviations from trend and is obtained from Hodrick-Prescott filter.

3 Conclusions

This paper quantitatively shows that a standard two sector open economy model can generate both a positive co-movement between two types of disaggregate investment and relatively high volatility of housing investment even without the introduction of land as an adjustment cost. Moreover, the open economy assumption that allows access to the international financial market helps generate a positive correlation between housing investment and house prices, a well-documented regularity that neoclassical growth models with a closed economy assumption fails to explain.

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Appendix A: Calibration

This appendix establishes a map between the production side of this model with that of DH. In other words, since the production technologies in this model can be considered as a reduced form of that in DH, we can establish a map between parameter specifications of the two model. In particular:

The output of intermediate good i at time t in DH is produced by the following technology:

$$x_{it} = k_{it}^{\theta_i}(z_{it}n_{it})^{1-\theta_i} \tag{3.28}$$

where $x_{it}, k_{it}, z_{it}, n_{it}$ are quantities of output of intermediate good, capital, productivity, and labor in intermediate sector i while $i \in \{b, m, s\}$ denote construction, manufactures, and services, respectively. This is equation (1) in DH.

The stochastic component of productivity shocks, $\tilde{z}_t$, associated with productivity z_t, follows and auto-regressive process as in the equation (6) in DH.

$$\tilde{z}_{t+1} = (log\ \tilde{z}_{b,t+1}, log\ \tilde{z}_{m,t+1}, log\ \tilde{z}_{s,t+1}) = B\tilde{z}_t + \epsilon_{t+1} \tag{3.29}$$

$y_{jt}, j \in \{c, d\}$, the quantity of final good, c, consumption and capital investment, and d, residential/housing investment, are produced using quantities b_{jt}, m_{jt}, s_{jt} of the three intermediate inputs as the equation (8) in DH:

$$y_{jt} = b_{jt}^{B_j} m_{jt}^{M_j} s_{jt}^{S_j} \tag{3.30}$$

where $S_j = 1 - B_j - M_J$

Using the intermediate goods market clearing conditions to substitute (1) into (8) of DH, we obtain the following reduced for final goods c:

$$y_{ct} = A_t^c (k_t^c)^{\alpha_c} (n_t^c)^{1-\alpha_c} \tag{3.31}$$

where $\alpha_c = \theta_b B_c + \theta_m M_c + \theta_s S_c$ and

$$log\ A_t^c = (1-\theta_b)B_c\ log\ z_{bt} + (1-\theta_m)M_c\ log\ z_{mt} + (1-\theta_s)S_c\ log\ z_{st} \tag{3.32}$$

Similarly for the residential sector, d:

$$y_{dt} = A_t^d (k_t^d)^{\alpha_d} (n_t^d)^{1-\alpha_d} \tag{3.33}$$

where $\alpha_d = \theta_b B_d + \theta_m M_d + \theta_s S_d$ and

$$log\ A_t^d = (1-\theta_b) B_d\ log\ z_{bt} + (1-\theta_m) M_d\ log\ z_{mt} + (1-\theta_s) S_d\ log\ z_{st} \tag{3.34}$$

We then can rewrite the productivity as:

$$\begin{bmatrix} log(A_{t+1}^c) \\ log(A_{t+1}^d) \end{bmatrix} = \begin{bmatrix} (1-\theta_b)B_c & (1-\theta_m)M_c & (1-\theta_s)S_c \\ (1-\theta_b)B_d & (1-\theta_m)M_d & (1-\theta_s)S_d \end{bmatrix} \begin{bmatrix} log\ z_{bt} \\ log\ z_{mt} \\ log\ z_{st} \end{bmatrix} \tag{3.35}$$

Or equivalently:

$$\tilde{a}_{t+1} = \begin{bmatrix} (1-\theta_b)B_c & (1-\theta_m)M_c & (1-\theta_s)S_c \\ (1-\theta_b)B_d & (1-\theta_m)M_d & (1-\theta_s)S_d \end{bmatrix} \tilde{z}_{t+1} = A\tilde{z}_{t+1} \tag{3.36}$$

Substitute the equation (6) of DH into the above equation we obtain the following reduced process for the productivity shocks:

$$\tilde{a}_{t+1} = \tilde{A}\tilde{a}_t + u_{t+1} \tag{3.37}$$

where $\tilde{A} = ABA^{-1}$ and $u_{t+1} = A\epsilon_{t+1}$

Using the values calculated by DH in the Table 3 and the Table 4, we can calculate the corresponding values for this paper as:$\alpha_c = 0.25, \alpha_h = 0.2$ and:

$$\tilde{A} = \begin{bmatrix} a_{cc} & a_{ch} \\ a_{hc} & a_{hh} \end{bmatrix} = \begin{bmatrix} 0.96 & -0.17 \\ 0.11 & 0.64 \end{bmatrix} \tag{3.38}$$

The covariance matrix of u_t is then equal to $\begin{bmatrix} 0.0135 & 0.0063 \\ 0.0063 & 0.0194 \end{bmatrix}$

Appendix B: Solution Method

I solve the models by the perturbation method.[13] Particularly the set of optimality conditions of the economy can be expressed as follows:

$$E_t\{F(Y_{t+1}, Y_t, X_{t+1}, X_t)\} = 0 \tag{3.39}$$

where Y_t is the vector of non-predetermined variables, and $X_t = [x_t^1, x_t^2]'$ is the state variable vector, x_t^1 are endogenous predetermined state variables while x_t^2 is exogenous state variables and follow an exogenous process given as:

$$x_{t+1}^2 = \Lambda x_t^2 + \tilde{\eta}\bar{\sigma}\epsilon_{t+1} \tag{3.40}$$

$\tilde{\eta}, \bar{\sigma}$ are given parameter. The solution of the optimal plan is of the form:

$$Y_t = g(X_t, \bar{\sigma}) \tag{3.41}$$

$$X_{t+1} = h(X_t, \bar{\sigma}) + \bar{\eta}\bar{\sigma}\epsilon_{t+1} \tag{3.42}$$

where $\bar{\eta} = [\emptyset, \tilde{\eta}]'$, these equations describe the policy and transition functions respectively. I compute a first order expansion of the two functions around the deterministic steady state.

[13] For more details, see Schmitt-Grohe and Uribe (2004)