Endogenous Biased Innovation, Demographic Heterogeneity, and Inequality[1]

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Abstract

Based on a three-factor, two-level constant elasticity of substitution production function, this paper develops an endogenous biased innovation model to analyze the effects of heterogeneity of population growth on income inequalities. Extending the innovation possibility frontier that includes skill-augmenting and unskilled-augmenting technical progress, we investigate the determination of skill-biased innovation and the dynamics of income inequality with the heterogeneity of population growth of skilled and unskilled labor in the neoclassical growth model. We show that the equilibrium skill-biased innovation can be introduced when the population growth of unskilled labor is larger than that of skilled labor. We also show that the shift of the innovation frontier can play significant role for growth and the relatively scarcity of skilled labor may produce labor income inequality in some relevant capital-skill complementarity. In the case where the frontier shifts with capital accumulation, for instance, the scarcity of skilled labor supply may provide more skill-biased innovation, thus promoting growth, but produce the labor income inequality. However, in the alternative case where the innovation frontier shifts with capital share, the scarcity of skilled labor supply may also provide more skill-biased innovation and labor income inequality, but produce growth stagnate. The implications of capital-augmenting technical progress on the dynamics of inequality are also investigated.

JEL: D33, E13, E25, O33

Keywords: skill-biased innovation, heterogeneity of population growth, capital-skill complementarity, inequality, neoclassical growth

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1. Introduction

Based on a three-factor, two-level constant elasticity of substitution production function,[2] we develop an endogenous biased innovation model to analyze the effects of heterogeneity of population growth on income inequalities. Recently, having analyzed the dynamics of technological unemployment in the framework of the induced factor-biased of innovation, Stiglitz (2014) suggested that the formulation of the induced skill-biased innovation is a promising research approach for analyzing the various inequalities in OECD countries.[3] One of the implications of the induced innovation framework in line with Kennedy (1964), Samuelson (1965), and Weizacker (1966) that relatively increasing the factor share can induce firms to introduce their own factor-augmenting technical progress in the maximization of the instantaneous cost reduction rate of change on the innovation frontier. In this framework, in the case of the innovation possibility frontier having a negative relationship between skill-augmenting and unskilled-augmenting technical progress, an increase in the income of skilled labor relative to unskilled labor can lead to skill-biased innovation. If the elasticity of substitution between skilled and unskilled labor is less than unity, the steady state is stable. However, this elasticity of substitution less than unity is not empirically plausible, and we need other formulation. Osumi (2019) examined the stability condition in this induced innovation framework whose innovation possibility frontier has a negative relationship between skill-augmenting and unskilled-augmenting technical progress in a three-factor, two-level constant elasticity of substitution production function that captures capital-skill complementarity in a Solow type neoclassical growth. This three-factor analysis shows that the stability condition is some empirically relevant capital-skill complementarity,[4] which represents an elasticity of substitution between capital and skilled labor less than one and an elasticity of substitution between capital and unskilled labor larger one. This implies that even in the elasticity of substitution between skilled and unskilled labor larger than unity, the smaller elasticity of substitution between capital and skilled labor make the steady state stable.

However, to characterize an endogenous skill-biased innovation in the stable steady state, we need two modifications in an induced biased innovation framework. First, we incorporate the heterogeneity of population growth of skilled labor and unskilled labor. If there is the homogenous case of population growth, no biased innovations are introduced at the steady state. In this case, the skill-biased technology occurs only in the transition process (Osumi; 2019). However, the heterogeneity of population growth can produce skill-biased innovations at the steady state. For

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[2] For pioneering works, see Griliches (1969) and Sato (1967).

[3] See Acemoglu (2002) and Hornstein et al. (2005). Recent studies have examined capital-augmenting technical progress as an improvement in automation and artificial intelligence technologies. See Acemoglu and Restrepo (2017, 2018a, 2018b), Kotlikoff and Sachs (2012), Korinek and Stiglitz (2017), and Graetz and Michaels (2018). In this paper, as an exogenous parameter, we analyze the impact of capital-augmenting technology on skill-biased innovation and income inequality.

[4] See Krusell et al. (2000) and Duffy et al. (2004).

instance, in the case where the population growth of unskilled labor is larger than that of skilled labor, the equilibrium skill-biased innovation can be introduced. Because the more scarcity of skilled labor makes profit maximizing firms to introduce more skill-biased innovation in the growth process. This heterogenous case of population growth is likely to occur if the new arrived technologies such as advancements of Information and Communication Technology (ICT) and Artificial Intelligence (AI) lead to the obsolescence of many types of skills. This implies an increasing number of unskilled workers, in other words, the more scarcity of skilled labor in the growing economy.

Second, we develop the innovation possibility frontier that has possible shifts. The weakness of the innovation possibility frontier in line with Kennedy (1964), Samuelson (1965), and Weizacker (1966) type is that the innovation frontier itself does not have the possibility of shifting.[5] This weakness uniquely both biased technologies and factor income ratio being constant at the steady state based on the formulation of the maximization of the instantaneous cost reduction rate of change on the concavity of the stationary innovation frontier. Therefore, at the steady state, skill-biased innovation and income inequality do not change unless the innovation frontier can shift or the induced innovation is derived from the alternative formulation.

In this paper, based on this induced innovation framework, we develop two possible shifts in the innovation possible frontier.[6] We first develop the induced innovation frontier incorporating the externality of capital accumulation that can expand outward at the innovation frontier.[7] R&D activity can be embodied in the new capital stock, and thus we take account of this property as a scale effect in possible shift of innovation frontier. Moreover, following Samuelson (1965), we alternatively develop the innovation frontier having the capital share as a positive shift parameter.[8] This implies that increasing capital share can enable profit maximizing firms to enlarge the resources devoted to R&D activity. This is referred to a profitability effect in possible shift of the innovation frontier. Thus, this effect enlarges the innovation possibility frontier to the outward. Developing these two modifications, our model can analyze the implications of the effects of R&D activities on the skill-biased innovation, growth and income inequality in the induced innovation framework.

Our main results are as follows. If population growth in unskilled labor is larger than that in skilled labor, skill-biased innovation can be introduced even at the steady state. In this case where the innovation frontier can shift with capital accumulation, the steady state is stable in some relevant capital-skill complementarity. A rising population of unskilled labor can promote growth because it has a positive influence on capital accumulation and therefore this expands the frontier

[5] See Nordhaus (1973) and Acemoglu (2015).

[6] Caselli and Coleman (2006) considered the same type of frontier in capital–skill complementarity. However, they did not analyze the induced biased innovation.

[7] See Adachi et al. (2019).

[8] Samuelson (1965) suggested that the capital share of income can be one of the shift parameters of the innovation possibility frontier. However, he did not have the model analysis.

outward. However, this relatively scarcity of skilled labor in growth process is likely to produce more skill-biased innovation and greater income inequality.[9] Alternatively, in the case where the innovation frontier shifts with capital share, we have different outcomes. In this case, in some relevant capital-skill complementarity with stable condition, rising population in unskilled labor produces declining the capital share. This declining capital share moves the innovation frontier inwards. Therefore, the growth rate is likely to be stagnate. However, since the relatively scarcity of skilled labor in the growth process can produce more skill-biased innovation, it may produce the more income inequality between skilled and unskilled labor.

The structure of this paper is as follows. Section 2 presents the basic model. Section 3 analyzes the dynamic system and the implications for the shift of induced skill-biased innovation and income inequality. Section 4 concludes.

2. Model

2.1 Three-factor production function

Following the framework in Osumi (2019), we develop the induced skill-biased innovation. We consider a three-factor production function that is twice differentiable and homogeneous of degree one:

$$Y = F(A_1 L_1, A_2 L_2, BK), \tag{1}$$

where Y is output, L_1 is skilled labor, L_2 is unskilled labor, K is capital stock, A_1 is skilled labor efficiency, A_2 is unskilled labor efficiency and B is capital efficiency. Based on the assumption that the three-factor production function is a weakly separable sub-aggregate production function, we specify a nested two-level CES production function that has two elasticity parameters: the elasticity of substitution between capital and skilled labor σ_1 and the elasticity of substitution between capital and unskilled labor σ_2:

$$Y = F(L_1, L_2, K)$$
$$= \left[[\delta_2\{\delta_1 L_1^{(1-\sigma 1)/\sigma 1} + (1-\delta_1)K\}^{(1-\sigma 1)/\sigma 1}\right]^{(1-\sigma 2)\sigma 1/(1-\sigma 1)\sigma 2} + (1-\delta_2)L_2^{(1-\sigma 2)/\sigma 2}\Big]^{\sigma 2/(1-\sigma 2)}. \tag{2}$$

In this specification, $\sigma_2 > \sigma_1$ provides a capital–skill complementarity technology[10] that has been widely estimated (Krusell et al., 2000; Hornstein et al., 2005), and we deal with this ongoing technical progress. In particular, our analysis focuses on inequalities in

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[9] For different approach, see Acemoglu (2010) and Korinek and Stiglitz (2017).

[10] Defining $c_{ij} \equiv F_{ij}F / F_i F_j$ as the partial elasticity of the complementarity between i and j, capital–skill complementarity is described as an inequality in which the elasticity of complementarity between capital and skilled labor is larger than that between capital and unskilled labor c_{1K} ($= F_{1K}F / F_1 F_K$) $>$ c_{2K} ($= F_{2K}F / F_2 F_K$). In our two-level CES production technology, $\sigma_2 > \sigma_1$ implies $c_{1K} > c_{2K}$ since $c_{1K} - c_{2K} = (\sigma_1^{-1} - \sigma_2^{-1})/(1 - F_2 L_2/F)$. Thus, $\sigma_2 > \sigma_1$ implies capital–skill complementarity. In addition, in our three-factor case, the elasticity of substitution is not always equal to the inverse of the elasticity of complementarity. Specifically, $c_{1K} = (\sigma_1^{-1} - \sigma_2^{-1})/(1 - F_2 L_2/F) + \sigma_2^{-1} \neq \sigma_1^{-1}$ although $c_{2K} = \sigma_2^{-1}$.

empirically relevant capital–skill complementarity $\sigma_2 > 1 > \sigma_1$.[11]

The assumption of constant returns to scale can describe the production function as follows:

$$Y = BKf(A_1L_1/BK, A_2L_2/BK) \cdot \quad (3)$$

We consider the long-run perfect competitive economy that has full employment. Let n_1 and n_2 denote the population growth of skilled labor and unskilled labor, respectively, and we assume that the population growth of unskilled labor is larger than that of skilled labor $n_2 > n_1$. In this setting, rewriting output per capital $y(= Y/K)$ leads to

$$y = Bf(\tilde{c}x/B, x/B), \quad (4)$$

where $\tilde{c}(\equiv A_1L_1/A_2L_2)$ is the effective skilled/unskilled labor ratio and $x(\equiv A_2L_2/K)$ is the effective unskilled labor/capital ratio. Over time, the movement of the induced skill-biased innovation represents the dynamics of $\tilde{c}$ and capital accumulation represents the dynamics of *x*.

In the long-run economy, the skilled wage w_1 and unskilled wage w_2 are competitively determined as their own marginal products. In this setting, the skilled wage rate w_1 and unskilled wage rate w_2 are given by $w_1 = \partial F/\partial L_1 = A_1 f_1$ and $w_2 = \partial F/\partial L_2 = A_2 f_2$, respectively. Therefore, labor income inequality $a/b(\equiv w_1L_1/w_2L_2)$, implying the labor share ratio, are written as follows:

$$a/b = f_1\tilde{c}/f_2, \quad (5)$$

where *a* is the skilled labor share and *b* is the unskilled labor share. On the other hands, the movement of wage inequality $\omega(\equiv w_1/w_2)$ becomes the difference of population growth between unskilled labor and skilled labor in the steady state.

2.2 Induced innovation frontier

Consider the induced biased technologies in line with the Kennedy (1964) and Samuelson (1965) type. We focus on two augmenting technologies, namely skill-augmenting technology $\alpha(\equiv \dot{A}_1/A_1)$ and unskilled-augmenting technology $\beta(\equiv \dot{A}_2/A_2)$, where the dot denotes d*x*/d*t*. Their rates of technical changes are given by the following innovation possibility frontier $q(\alpha,\beta,g)=0$, which is rewritten as

$$\beta = \beta(\alpha, g), \qquad \beta_\alpha < 0,\ \beta_{\alpha\alpha} < 0,\ \beta_g > 0,\ \beta_{\alpha g} > 0. \quad (6)$$

Here, $\beta_\alpha < 0$ and $\beta_{\alpha\alpha} < 0$ exhibit the concavity of the innovation frontier that implies the resource constraints devoted to these factor-biased technologies. $g(\equiv \dot{K}/K)$ expresses capital accumulation and $\beta_g > 0$ represents a shift in the innovation possibility frontier. Following Adachi et al. (2019), we first assume that the innovation frontier shifts with capital accumulation. Later, we discuss the implication of the alternative shift parameter, that is capital share. $\beta_g > 0$ implies that R&D activity is embodied in the newly capital stock, thus the capital accumulation can produce more possible innovation. We formulate this as the

[11] See Duffy et al. (2004) and Hornstein et al. (2005).

scale effect of capital accumulation on the innovation frontier. $\beta_{\alpha g} > 0$ implies that the equilibrium skill-augmenting technology is an increasing function of capital accumulation. Then, representative firms facing the innovation frontier aim to maximize the instantaneous cost reduction rate of change $a\alpha + b\beta(\alpha, g)$ with respect to α, where a is the skilled labor share and b is the unskilled labor share. Solving this maximization problem yields $-\beta_\alpha(\alpha, g) = a/b$. This means that the tangency of the innovation possibility curve equivalent to the skilled/unskilled labor share ratio, implying income inequality, determines the equilibrium skill-biased innovation. This is explicitly shown as

$$-\beta_\alpha(\alpha, g) = \frac{f_1(\tilde{c}x/B, x/B)\tilde{c}}{f_2(\tilde{c}x/B, x/B)}, \tag{7}$$

where

$$a = \frac{f_1(\tilde{c}x/B, x/B)\tilde{c}x/B}{f(\tilde{c}x/B, x/B)}, \tag{8}$$

$$b = \frac{f_2(\tilde{c}x/B, x/B)x/B}{f(\tilde{c}x/B, x/B)}. \tag{9}$$

Equation (7) is solved for each equilibrium biased innovation α^* and β^* as follows.

$$\alpha^* = \alpha(x, \tilde{c}, g, B), \tag{10a}$$

$$\beta^* = \beta[\alpha(x, \tilde{c}, g, B), g] = \beta(x, \tilde{c}, g, B). \tag{10b}$$

2.3 Dynamics

We consider a standard Solow type neoclassical growth model. In this model, aggregate savings determine investment and thus can provide capital accumulation. Then, the rate of change of the effective unskilled labor/capital ratio $\dot{x}/x$ and that of the effective skilled/unskilled labor ratio $\dot{\tilde{c}}/\tilde{c}$ and capital accumulation are respectively given by

$$\dot{x}/x = \beta + n_2 - sBf(\tilde{c}x/B, x/B), \tag{11}$$

$$\dot{\tilde{c}}/\tilde{c} = \alpha(x, \tilde{c}, g, B) - \beta(x, \tilde{c}, g, B) + n_1 - n_2, \tag{12}$$

$$g = sBf(\tilde{c}x/B, x/B), \tag{13}$$

where s represents the saving rate assumed to be simply constant. Then, the steady state is given by the solution to

$$\beta(x^*, \tilde{c}^*, g^*, B) + n_2 = sBf(\tilde{c}^* x^*/B, x^*/B), \tag{14}$$

$$\alpha(x^*, \tilde{c}^*, g^*, B) + n_1 = \beta(x^*, \tilde{c}^*, g^*, B) + n_2, \tag{15}$$

$$g^* = sBf(\tilde{c}^* x^*/B, x^*/B). \tag{16}$$

From these equations, we have the following proposition.

Proposition 1

In the induced innovation framework in line with Kennedy and Samuelson type, if the population growth of unskilled labor is larger than that of skilled labor, the equilibrium skill-biased innovation can be introduced at the steady state.

[Insert Figure 1]

Figure 1 shows this proposition. Note that the abundance of unskilled labor, in other words, the scarcity of skilled labor in growing economy can induce firms to promote more skill-augmenting technology. [12] Moreover, purely skill-biased innovation can appear if the population growth of unskilled labor is high such that the upward line satisfying $\alpha + n_1 = \beta + n_2$ intersects with point α_0 on the innovation frontier in Figure 1. However, we also note that each population growth in skilled and unskilled labor is the same, we have no biased innovation the steady state. In the next section, we analyze the properties of the dynamics and effects on the induced bias and inequalities.

3. Analysis

We first examine the stability of the dynamics and then analyze the comparative statics of income inequality, the induced bias innovation, and growth at the steady state. Finally, we consider the implication of the shift of innovation possibility frontier.

3.1 Stability

We start by examining the properties of the equilibrium biased technologies. In equation (7), totally differentiating with respect to α, x, $\tilde{c}$, g, and B yields the following equation in our specified two-level CES production function: [13]

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[12] Contrary to this case, Acemoglu (1998) formulates the abundance of skilled labor producing the skill-biased technology. This effect is referred to a market size effect.

[13] Total differentiation yields

$$(\beta_{\alpha\alpha}\alpha / \beta_{\alpha})\hat{\alpha} = (f_{11}l_1 / f_1 + f_{12}l_2 / f_1 - f_{21}l_1 / f_2 - f_{22}l_2 / f_2)(\hat{x} - \hat{B}) + (f_{11}l_1 / f_1 - f_{21}l_1 / f_1 + 1)\hat{\tilde{c}}$$
$$-(\beta_{\alpha g} g / \beta_{\alpha})\hat{g}$$

where $l_1 \equiv A_1 L_1 / BK, l_2 \equiv A_2 L_2 / BK$. In our weakly separable two-level CES production technology, $f_{ij}l_i / f_j (i, j = 1,2)$ are specified as follows: $f_{11}l_1 / f_1 = -(\kappa\sigma_1^{-1} + ab\sigma_2^{-1}) / (1-b) < 0$, $f_{12}l_2 / f_1 = b\sigma_2^{-1} > 0$, $f_{21}l_1 / f_2 = a\sigma_2^{-1} > 0$, and $f_{22}l_2 / f_2 = -(1-b)\sigma_2^{-1} < 0$. These specifications lead to equation (17).

$$(\beta_{\alpha\alpha}\alpha / \beta_{\alpha})\hat{\alpha} = \frac{\kappa}{1-b}(\sigma_2{}^{-1} - \sigma_1{}^{-1})(\hat{x} - \hat{B}) + \frac{1}{1-b}(-a\sigma_2{}^{-1} - \kappa\sigma_1{}^{-1} + a + \kappa)\hat{\tilde{c}} - (\beta_{\alpha g} g / \beta_{\alpha})\hat{g} \qquad (17)$$

where $\hat{x}(\equiv dx / x)$ denotes the percentage change in x and $\kappa(=1-a-b)$ represents the capital share. Therefore, assuming capital–skill complementarity technology ($\sigma_2 > \sigma_1$), we find the effect of these parameters on the equilibrium skill-biased innovation in the elasticity form as follows: [14]

$$\alpha_x x / \alpha = -\alpha_B B / \alpha < 0, \quad \alpha_c \tilde{c} / \alpha \gtrless 0, \quad \alpha_g g / \alpha > 0 \qquad (18)$$

Similarly, we find the effect of each parameter on the equilibrium unskilled-biased innovation in the elasticity form.[15] Here, an increase in x leads to a decrease in α. However, increases in g and B lead to an increase in α. A similar result applies to the effect of $\tilde{c}$, although this effect is ambiguous. However, the increase in $\tilde{c}$ can sufficiently lead to a decrease in α if the stability condition in (21) is satisfied. This stability condition corresponds to an elasticity of substitution less than unity in the two-factor case. Moreover, we have a positive effect of g on α and β if $\tilde{\beta}_g = \beta_\alpha \alpha_g + \beta_g > 0$. These expansion effects of capital accumulation can play significant roles in the comparative statics of inequalities at the steady state.

We consider the stability condition of the dynamics. Linearizing in equations (11) and (12) that incorporates (13) at the steady state and rearranging, we have the following dynamic matrix equation:

$$\begin{pmatrix} \delta(\dot{x}/x)/\delta x \\ \delta(\dot{\tilde{c}}/\tilde{c})/\delta\tilde{c} \end{pmatrix} = \begin{pmatrix} \beta_\alpha \alpha(\alpha_x x/\alpha) + A(a+b)g & \beta_\alpha \alpha(\alpha_c \tilde{c}/\alpha) + Aag \\ (1-\beta_\alpha)\alpha(\alpha_x x/\alpha) + B(a+b)g & (1-\beta_\alpha)\alpha(\alpha_c \tilde{c}/\alpha) + Bag \end{pmatrix} \begin{pmatrix} dx/x \\ d\tilde{c}/\tilde{c} \end{pmatrix} \qquad (19)$$

where

$$A = \tilde{\beta}_g - 1, \quad B = \alpha_g - \tilde{\beta}_g, \quad \tilde{\beta}_g = \beta_\alpha \alpha_g + \beta_g. \qquad (20)$$

Describing the matrix of the partial derivatives of the differential equations as J, the stability of the steady state is then locally satisfied when the trace of the matrix J is negative and the

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[14] The effects of each parameter on the equilibrium skilled-biased innovation are given as follows: $\alpha_x x / \alpha = -\alpha_B B / \alpha = \frac{\beta_\alpha}{\beta_{\alpha\alpha}\alpha}\frac{\kappa}{1-b}(\sigma_2{}^{-1} - \sigma_1{}^{-1}) < 0$,

$$\alpha_c \tilde{c} / \alpha = \frac{\beta_\alpha}{\beta_{\alpha\alpha}\alpha}\frac{1}{1-b}(-a\sigma_2{}^{-1} - \kappa\sigma_1{}^{-1} + a + \kappa), \quad \alpha_g g / \alpha = \frac{-\beta_{\alpha g}}{\beta_{\alpha\alpha}\alpha} g > 0.$$

[15] The effects of each parameter on the equilibrium unskilled-biased innovation are given as follows: $\beta_x x / \beta = -\beta_B B / \beta = \beta_\alpha \alpha_B B / \beta\beta_\alpha \alpha_x x / \beta > 0$, $\beta_c \tilde{c} / \beta = \beta_\alpha \alpha_c \tilde{c} / \beta$, $\tilde{\beta}_g g / \beta = (\beta_\alpha \alpha_g + \beta_g) g / \beta$.

determinant of the matrix J is positive. [16] From these conditions, we have the following proposition about stability at the steady state.

Proposition 2

With endogenous factor-biased innovation that may shift with capital accumulation in the three-factor, two-level CES production economy as well as relatively large population growth in unskilled labor, the steady state is sufficiently stable if the expansion effect of the innovation frontier is weak and there is some relevant capital–skill complementarity,

$$1-\beta_\alpha-\beta_g>0\,,\quad a\sigma_2^{-1}+b\kappa\sigma_1^{-1}-a-b\kappa>0\,. \tag{21}$$

[Insert Figure 2]

Figures 2 shows Proposition 2 in the case of $1-\beta_\alpha-\beta_g>0$. We make remarks. First, the stability condition is the same as that in the case of the homogenous population growth.[17] Thus, the population growth does not matter in the stability of the dynamical system. Second, in the three-factor case, the steady state is stable in some empirically relevant capital–skill complementarity $\sigma_2>1>\sigma_1$ as well as $1>\sigma_2>\sigma_1$. The former implies that even if there is large substitutability between skilled and unskilled labor, implying $\sigma_2>1$, which is the empirically relevant case,[18] the complementarity between capital and skilled labor makes the steady state stable.

3.2 Comparative statics

We first consider the case of the effective unskilled labor/capital ratio and effective skilled/unskilled labor ratio. Then, we analyze the effects of the parameters on the labor shares, inequality, and biased technologies at the steady state. Total differentiation in the steady state produces the following matrix:

$$\begin{pmatrix} \beta_\alpha\alpha(\alpha_x x/\alpha)+A(a+b)g & \beta_\alpha\alpha(\alpha_c\tilde{c}/\alpha)+Aag \\ (1-\beta_\alpha)\alpha(\alpha_x x/\alpha)+B(a+b)g & (1-\beta_\alpha)\alpha(\alpha_c\tilde{c}/\alpha)+Bag \end{pmatrix}\begin{pmatrix} dx/x \\ d\tilde{c}/\tilde{c} \end{pmatrix}=\begin{pmatrix} -A \\ -B \end{pmatrix}ds/s$$

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[16] With some calculation, the trace and determinant of the matrix are respectively given by

$$trJ=-\left[\frac{\beta_\alpha}{\beta_{\alpha\alpha}}\frac{1}{b(1-b)}(a\sigma_2^{-1}+b\kappa\sigma_1^{-1}-a-b\kappa)+b(1-\beta_\alpha-\beta_g)g\right]<0\,,$$

$$\det J\equiv\Delta=\frac{\beta_\alpha}{\beta_{\alpha\alpha}}(1-\beta_\alpha-\beta_g)g\frac{1}{1-b}(a\sigma_2^{-1}+b\kappa\sigma_1^{-1}-a-b\kappa)>0\,.$$

[17] See Osumi (2019).

[18] See Ciccone and Peri (2005). In the two-factor case, if the elasticity of substitution between skilled and unskilled labor is larger than unity, the steady state may always be unstable.

$$+\begin{pmatrix} -\beta_\alpha \alpha(\alpha_B B/\alpha) - A\kappa g \\ -(1-\beta_\alpha)\alpha(\alpha_B B/\alpha) - B\kappa g \end{pmatrix} dB/B + \begin{pmatrix} 0 \\ -n_1 \end{pmatrix} dn_1/n_1 + \begin{pmatrix} -n_2 \\ n_2 \end{pmatrix} dn_2/n_2 \cdot \quad (22)$$

Assuming that the stability condition $\Delta > 0$ is satisfied. we obtain the following main results for capital–skill complementarity technology: [19]

$$\hat{x}/\hat{s} < 0, \quad \hat{\tilde{c}}/\hat{s} > 0, \quad (23a)$$

$$\hat{x}/\hat{B} < 0, \quad \hat{\tilde{c}}/\hat{B} > 0, \quad (23b)$$

$$\hat{x}/\hat{n}_2 \gtreqless 0, \quad \hat{\tilde{c}}/\hat{n}_2 \gtreqless 0. \quad (23c)$$

Thus, both the increase in the saving rate and the advancement of capital-augmenting technical progress lead to a decrease in the effective unskilled labor/capital ratio, but an increase in the effective skill/unskilled labor ratio. Alternatively, the effects of population growth of unskilled labor on these two equilibrium variables are ambiguous.

From these results, we can analyze the effects of the parameters on the labor shares, inequality, and biased technologies at the steady state. Table 1 summarizes the results. Total differentiation with respect to the skilled labor share a, unskilled labor share b, and aggregate labor share $s_L(=a+b)$ in our specified production function provides the following equations: [20]

$$\hat{a} = \frac{\kappa}{1-b}(b\sigma_2^{-1} - \sigma_1^{-1} + 1 - b)(\hat{x} - \hat{B}) + \frac{1}{1-b}(-\kappa\sigma_1^{-1} - ab\sigma_2^{-1} + \kappa + ab)\hat{\tilde{c}}, \quad (24)$$

$$\hat{b} = \kappa(1-\sigma_2^{-1})(\hat{x} - \hat{B}) + a(\sigma_2^{-1} - 1)\hat{\tilde{c}}, \quad (25)$$

$$\hat{s}_L = \frac{a}{a+b}\hat{a} + \frac{b}{a+b}\hat{b} \cdot \quad (26)$$

Calculating the above equations, we have the following consequences in some empirically relevant capital–skill complementarity $(\sigma_2 > 1 > \sigma_1)$. Concerning the saving rate and capital-augmenting technical progress, we find the same results because the frontier does not shift at the steady state:

$$\hat{a}/\hat{s} = \hat{b}/\hat{s} = \hat{s}_L/\hat{s} = \hat{a}/\hat{B} = \hat{b}/\hat{B} = \hat{s}_L/\hat{B} = \frac{1}{\Delta}(1-\beta_\alpha-\beta_g)\frac{\beta_\alpha}{\beta_{\alpha\alpha}}\kappa(1-\sigma_2^{-1})(1-\sigma_1^{-1}) < 0, \quad (27)$$

$$\hat{\alpha}/\hat{s} = \hat{\beta}/\hat{s} = 0, \quad \hat{\alpha}/\hat{B} = \hat{\beta}/\hat{B} = 0 \cdot \quad (28)$$

Both the increase in saving rate and the advancement of capital-augmenting technical progress decrease the skilled labor shares, unskilled labor share, and aggregate labor share at the same rates. Thus, the capital shares rise, but labor income inequality a/b does not change. These results come from the constancy of capital accumulation at the steady state. Thus, the stationary innovation frontier provides the constancy of the labor income ratio. Hence, the movement of the

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[19] The details are given by the appendix 1.

[20] Total differentiation yields

$$\hat{a} = (f_{11}l_1/f_1 + f_{12}l_2/f_1 + 1 - f_1l_1/f - f_2l_2/f)(\hat{x} - \hat{B}) + (1 + f_{11}l_1/f_1 - f_1l_1/f)\hat{c},$$

$$\hat{b} = (f_{21}l_1/f_2 + f_{22}l_2/f_1 + 1 - f_1l_1/f - f_2l_2/f)(\hat{x} - \hat{B}) + (f_{21}l_1/f_2 - f_1l_1/f)\hat{c} \cdot$$

skilled labor shares can synchronize with that of the unskilled labor share and unbiased innovation occurs at steady state.

However, the consequences of population growth provide different outcomes because the capital accumulation changes and thus the innovation can shift. Therefore, the positive shift in the innovation frontier has a positive influence on growth, which may affect income inequality. The effects of the population growth in unskilled labor provide the following results: [21]

$$\hat{a}/\hat{n}_2 = \frac{1}{\Delta} n_2 \left[-\frac{\beta_\alpha}{\beta_{\alpha\alpha}} \kappa (1-\sigma_2^{-1})(1-\sigma_1^{-1}) + (1-\alpha_g) \frac{b}{1-b} (a\sigma_2^{-1} + \kappa\sigma_1^{-1} - a - \kappa) g \right], \quad (29)$$

$$\hat{b}/\hat{n}_2 = \frac{1}{\Delta} n_2 (1-\sigma_2^{-1}) \left[-\frac{\beta_\alpha}{\beta_{\alpha\alpha}} \kappa (1-\sigma_1^{-1}) + (1-\alpha_g) a g \right], \quad (30)$$

$$\hat{s}_L/\hat{n}_2 = \frac{1}{\Delta} n_2 \kappa \left[-\frac{\beta_\alpha}{\beta_{\alpha\alpha}} (1-\sigma_2^{-1})(1-\sigma_1^{-1}) + (1-\alpha_g) \frac{b}{1-b} \frac{a}{1-\kappa} (\sigma_1^{-1} - \sigma_2^{-1}) g \right], \quad (31)$$

$$\hat{a}/\hat{n}_2 - \hat{b}/\hat{n}_2 = \frac{1-\alpha_g}{1-\beta_\alpha-\beta_g} \frac{\beta_{\alpha\alpha}}{\beta_\alpha} n_2, \quad (32)$$

$$\hat{\alpha}/\hat{n}_2 = \frac{1}{1-\beta_\alpha-\beta_g} \frac{n_2}{\alpha} > 0, \quad (33)$$

$$\hat{\beta}/\hat{n}_2 = \frac{\beta_\alpha+\beta_g}{1-\beta_\alpha-\beta_g} \frac{n_2}{\beta}. \quad (34)$$

Moreover, we can have the results in the case of homogenous population growth $n(= n_1 = n_2)$ as follows: [22]

$$\hat{a}/\hat{n} = \frac{-1}{\Delta} n \left[\frac{\beta_\alpha}{\beta_{\alpha\alpha}} (1-\beta_\alpha) \kappa (1-\sigma_2^{-1})(1-\sigma_1^{-1}) + (\alpha_g - \tilde{\beta}_g) \frac{b}{1-b} (a\sigma_2^{-1} + \kappa\sigma_1^{-1} - a - \kappa) g \right] \quad (35)$$

$$\hat{b}/\hat{n} = \frac{-1}{\Delta} n (1-\sigma_2^{-1}) \left[\frac{\beta_\alpha}{\beta_{\alpha\alpha}} (1-\beta_\alpha) \kappa (1-\sigma_1^{-1}) + (\alpha_g - \tilde{\beta}_g) a g \right], \quad (36)$$

$$\hat{s}_L/\hat{n} = \frac{-1}{\Delta} n \kappa \left[\frac{\beta_\alpha}{\beta_{\alpha\alpha}} (1-\beta_\alpha)(1-\sigma_2^{-1})(1-\sigma_1^{-1}) + (\alpha_g - \tilde{\beta}_g) \frac{b}{1-b} \frac{a}{1-\kappa} (\sigma_1^{-1} - \sigma_2^{-1}) g \right], \quad (37)$$

$$\hat{a}/\hat{n} - \hat{b}/\hat{n} = \frac{-1}{1-\beta_\alpha-\beta_g} \frac{\beta_{\alpha\alpha}}{\beta_\alpha} (\alpha_g - \tilde{\beta}_g) n, \quad (38)$$

$$\hat{\alpha}/\hat{n} = \hat{\beta}/\hat{n} = \frac{\beta_g}{1-\beta_\alpha-\beta_g} \frac{n}{\beta} > 0 \quad (39)$$

Summarizing the outcomes, we have the following proposition.

Proposition 3

With endogenous factor-biased innovation that may shift with capital accumulation in the three-

[21] For the case of homogenous population growth $n(= n_1 = n_2)$, see Osumi (2019).

[22] See Osumi (2019).

factor case, with some relevant capital–skill complementarity technology as well as relatively large population growth in unskilled labor, the following holds in the steady state:

1. *The increase in the saving rate and the advancement of capital-augmenting technology cannot move the innovation possibility frontier, and thus cannot promote growth. Because they cannot increase the capital accumulation. However, aggregate labor income share decreases although labor income inequality does not change.*
2. *However, the more increase in the population growth of unskilled labor can lead to expanding the innovation possibility frontier, thus resulting in more growth, more skill-biased innovation, and more labor income inequality if the expansion effect of skill-biased innovation is small.*
3. *If the population growth of unskilled labor is the same as that of skilled labor, owing to decreasing the capital accumulation, population decline may move the innovation possibility frontier inward, thus lead to decreasing growth, but more labor income inequality and decreasing aggregate labor shares if the expansion effect of skill-biased innovation is larger than that of unskilled-biased innovation.*

Table 1. Effects of the parameters on labor shares, inequality, and biased innovation in some relevant capital-skill complementarity $\sigma_2 > 1 > \sigma_1$: The Case $\beta = \beta(\alpha, g)$

	χ	$\tilde{c}$	a	b	s_L	a/b	g	α	β
s	−	+	−	−	−	0	0	0	0
B	−	+	−	−	−	0	0	0	0
n_2	$+^a$	$-^b$	$+^a$	$+^a$	$+^a$	$+^a$	+	+	$+^c$
$n(=n_1=n_2)$	$+^e$	$-^f$	$+^e$	$+^e$	$+^e$	$-^g$	+	+	+

Note: $1-\beta_\alpha-\beta_g>0$, $a\sigma_2^{-1}+b\kappa\sigma_1^{-1}-a-b\kappa>0$

$a>0, b<0$ if $1>\alpha_g$, $c>0$ if $\beta_\alpha+\beta_g>0$

$e>0$, $f<0$ if $\alpha_g \cong \tilde{\beta}_g$, $g\le 0$ if $\alpha_g \ge \tilde{\beta}_g$

[Insert Figure 3]

Figures 3 shows the effect of population increase in the unskilled labor on the innovation possibility frontier. Note that we obtain our results when the population of skilled labor is relatively scarcity and the innovation possibility frontier is rather stationary. Therefore, if skilled labor supply is increasing, and the expansion effect of the innovation frontier is large, some outcomes may be reversed. Also, our outcomes depend on the structure of relevant capital-skill complementarity. Thus, if the production structure changes, the outcomes may change. Furthermore, if the shift of innovation possibility frontier is alternative one, we may have the different outcomes. We examine the implication of shift of the innovation frontier.

3.3 Alternative shift of the innovation possibility frontier

Finally, we analyze the effect of capital share on the shift of innovation possibility frontier and its influence on income inequalities and biased innovations. Larger profitability can enlarge resources devoted to more investing to the R&D activity, and thus has a positive influence on the innovation frontier. Samuelson (1965) suggested the capital share as the shift of innovation possibility frontier.[23] We consider this case as a profitability effect.

In this case, the innovation frontier exhibits

$$\beta = \beta(\alpha, \kappa), \qquad \beta_\alpha < 0,\ \beta_{\alpha\alpha} < 0,\ \beta_\kappa > 0,\ \beta_{\alpha\kappa} > 0, \tag{40}$$

where κ is capital share. Since the formulation of factor-biased innovation is the same, the solution to the maximization of the instantaneous cost reduction rate of change on the concavity of the innovation possibility frontier yields $-\beta_\alpha(\alpha,\kappa) = a/b$, which is shown as $-\beta_\alpha(\alpha,\kappa) = \dfrac{f_1(\tilde{c}x/B, x/B)\tilde{c}}{f_2(\tilde{c}x/B, x/B)}$. Therefore, we have $\alpha^* = \alpha(x,\tilde{c},\kappa,B)$, $\beta^* = \beta[\alpha(x,\tilde{c},\kappa,B),\kappa] = \beta(x,\tilde{c},\kappa,B)$. Accordingly, as the growth dynamics are also the same, these dynamics are given by

$$\dot{x}/x = \beta(x,\tilde{c},\kappa,B) + n_2 - sBf(\tilde{c}x/B, x/B), \tag{41}$$

$$\dot{\tilde{c}}/\tilde{c} = \alpha(x,\tilde{c},\kappa,B) - \beta(x,\tilde{c},\kappa,B) + n_1 - n_2, \tag{42}$$

$$\kappa = 1 - \frac{f_1(\tilde{c}x/B, x/B)\tilde{c}x/B}{f(\tilde{c}x/B, x/B)} - \frac{f_2(\tilde{c}x/B, x/B)x/B}{f(\tilde{c}x/B, x/B)}. \tag{43}$$

Hence, the steady state is given by the solution to

$$\beta(x^*,\tilde{c}^*,\kappa^*,B) + n_2 = sBf(\tilde{c}^*x^*/B, x^*/B), \tag{44}$$

$$\alpha(x^*,\tilde{c}^*,\kappa^*,B) + n_1 = \beta(x^*,\tilde{c}^*,\kappa^*,B) + n_2, \tag{45}$$

$$\kappa^* = 1 - \frac{f_1(\tilde{c}^*x^*/B, x^*/B)\tilde{c}^*x^*/B}{f(\tilde{c}^*x^*/B, x^*/B)} - \frac{f_2(\tilde{c}^*x^*/B, x^*/B)x^*/B}{f(\tilde{c}^*x^*/B, x^*/B)}. \tag{46}$$

Total differentiation in the steady state produces the following matrix:

$$\begin{pmatrix} \beta_\alpha\alpha(\alpha_x x/\alpha) + \tilde{\beta}_\kappa\kappa(\kappa_x x/\kappa) - (a+b)g & \beta_\alpha\alpha(\alpha_c\tilde{c}/\alpha) + \tilde{\beta}_\kappa\kappa(\kappa_c\tilde{c}/\kappa) - ag \\ (1-\beta_\alpha)\alpha(\alpha_x x/\alpha) + (\alpha_\kappa - \tilde{\beta}_\kappa)\kappa(\kappa_x x/\kappa) & (1-\beta_\alpha)\alpha(\alpha_c\tilde{c}/\alpha) + (\alpha_\kappa - \tilde{\beta}_\kappa)\kappa(\kappa_c\tilde{c}/\kappa) \end{pmatrix} \begin{pmatrix} dx/x \\ d\tilde{c}/\tilde{c} \end{pmatrix}$$

[23] For detail analysis, see Acemoglu (2015).

$$= \begin{pmatrix} g \\ 0 \end{pmatrix} ds/s + \begin{pmatrix} -\beta_\alpha \alpha(\alpha_B B/\alpha) - \tilde{\beta}_\kappa \kappa(\kappa_B B/\kappa) + \kappa g \\ -(1-\beta_\alpha)\alpha(\alpha_B B/\alpha) - (\alpha_\kappa - \tilde{\beta}_\kappa)\kappa(\kappa_B B/\kappa) \end{pmatrix} dB/B$$

$$+ \begin{pmatrix} 0 \\ -n_1 \end{pmatrix} dn_1/n_1 + \begin{pmatrix} -n_2 \\ n_2 \end{pmatrix} dn_2/n_2 . \qquad (47)$$

where

$$\tilde{\beta}_\kappa = \beta_\alpha \alpha_\kappa + \beta_\kappa . \qquad (48)$$

The dynamic system is stable if the following determinant $\tilde{\Delta}$ of this system is locally positive. From this condition, we have the following proposition about stability at the steady state.

Proposition 4

With endogenous factor-biased innovation that may shift with capital share in the three-factor, two-level CES production economy as well as relatively large population growth in unskilled labor, the steady state is sufficiently stable if the following inequality is satisfied in some relevant capital–skill complementarity,

$$-\frac{\beta_\kappa \beta_\alpha}{\beta_{\alpha\alpha}} \kappa(1-\kappa)(1-\sigma_1{}^{-1})(1-\sigma_2{}^{-1})$$

$$+\frac{g}{1-b}\frac{(1-\beta_\alpha)\beta_\alpha}{\beta_{\alpha\alpha}}(b\kappa\sigma_1{}^{-1} + a\sigma_2{}^{-1} - b\kappa - a) + (\alpha_\kappa - \tilde{\beta}_\kappa)\kappa g \frac{ab}{1-b}(\sigma_2{}^{-1} - \sigma_1{}^{-1}) > 0 . \qquad (49)$$

[Insert Figure 4]

Figure 4 shows the stability region. Intuitively, the steady state is stable in some empirically relevant capital–skill complementarity $\sigma_2 > 1 > \sigma_1$ as well as $1 > \sigma_2 > \sigma_1$ if the difference between the expansion effect of skill-biased innovation α_κ and that of unskilled-biased innovation $\tilde{\beta}_\kappa$ is small. Note that this difference between α_κ and $\tilde{\beta}_\kappa$ is smaller, the stability region can be enlarged, implying that the stability is more satisfied in some empirically relevant capital–skill complementarity $\sigma_2 > 1 > \sigma_1$.

Assuming the stability condition, we have the consequences of the parameters at the steady state. [24] Table 2 summarizes the effects on labor shares, inequalities and biased innovations. Calculating the comparative statics provides the following main results: Contrary to the case where the innovation frontier with capital accumulation, this case enables the capital-augmenting technology to have positive influence on the frontier, and thus produces more skill-biased innovation. Moreover, we have the opposite effects of population growth. Because the outcomes of the effect of capital share on the shift of innovation possibility frontier are different from those

[24] For the detail analysis, see Osumi (2020).

of the effects of capital accumulation on the shift of innovation possibility frontier.

$$\hat{a}/\hat{s}=\hat{a}/\hat{B}=\frac{1}{\tilde{\Delta}}g\kappa\{(1-\beta_\alpha)\frac{\beta_\alpha}{\beta_{\alpha\alpha}}+(\alpha_\kappa-\tilde{\beta}_\kappa)b\}(1-\sigma_2^{-1})(1-\sigma_1^{-1})<0, \tag{50}$$

$$\hat{b}/\hat{s}=\hat{b}/\hat{B}=\frac{1}{\tilde{\Delta}}g\kappa\{(1-\beta_\alpha)\frac{\beta_\alpha}{\beta_{\alpha\alpha}}-(\alpha_\kappa-\tilde{\beta}_\kappa)a\}(1-\sigma_2^{-1})(1-\sigma_1^{-1}), \tag{51}$$

$$\hat{s}_L/\hat{s}=\hat{s}_L/\hat{B}=\frac{1}{\tilde{\Delta}}g\kappa(1-\beta_\alpha)\frac{\beta_\alpha}{\beta_{\alpha\alpha}}(1-\sigma_2^{-1})(1-\sigma_1^{-1})<0, \tag{52}$$

$$\hat{a}/\hat{s}-\hat{b}/\hat{s}=\hat{a}/\hat{B}-\hat{b}/\hat{B}=\frac{1}{\tilde{\Delta}}g\kappa(\alpha_\kappa-\tilde{\beta}_\kappa)(a+b)(1-\sigma_2^{-1})(1-\sigma_1^{-1})<0, \tag{53}$$

$$\hat{\alpha}/\hat{s}=\hat{\alpha}/\hat{B}=\frac{-1}{\tilde{\Delta}}g\frac{\beta_\alpha}{\beta_{\alpha\alpha}\alpha}\beta_\kappa\kappa(1-\kappa)(1-\sigma_2^{-1})(1-\sigma_1^{-1})>0, \tag{54}$$

$$\hat{\beta}/\hat{s}=\hat{\beta}/\hat{B}=\frac{-1}{\tilde{\Delta}}g\frac{\beta_\alpha}{\beta_{\alpha\alpha}\beta}\beta_\kappa\kappa(1-\kappa)(1-\sigma_2^{-1})(1-\sigma_1^{-1})>0, \tag{55}$$

$$\hat{a}/\hat{n}_2=\frac{1}{\tilde{\Delta}}n_2\left[-(\frac{\beta_\alpha}{\beta_{\alpha\alpha}}+\alpha_\kappa b)\kappa(1-\sigma_2^{-1})(1-\sigma_1^{-1})+\frac{b}{1-b}(a\sigma_2^{-1}+\kappa\sigma_1^{-1}-a-\kappa)g\right]>0, \tag{56}$$

$$\hat{b}/\hat{n}_2=\frac{1}{\tilde{\Delta}}n_2(1-\sigma_2^{-1})\left[-(\frac{\beta_\alpha}{\beta_{\alpha\alpha}}+\alpha_\kappa a)\kappa(1-\sigma_1^{-1})+ag\right], \tag{57}$$

$$\hat{s}_L/\hat{n}_2=\frac{1}{\tilde{\Delta}}n_2\kappa\left[-\frac{\beta_\alpha}{\beta_{\alpha\alpha}}(1-\sigma_2^{-1})(1-\sigma_1^{-1})+\frac{a}{1-\kappa}\frac{b}{1-b}(\sigma_1^{-1}-\sigma_2^{-1})g\right]>0, \tag{58}$$

$$\hat{a}/\hat{n}_2-\hat{b}/\hat{n}_2=\frac{1}{\tilde{\Delta}}n_2\left[-\alpha_\kappa(a+b)\kappa(1-\sigma_2^{-1})(1-\sigma_1^{-1})+\frac{1}{1-b}(a\sigma_2^{-1}+b\kappa\sigma_1^{-1}-a-b\kappa)g\right]>0 \tag{59}$$

$$\hat{\alpha}/\hat{n}_2=\frac{1}{\tilde{\Delta}}\frac{n_2}{\alpha}\frac{g}{1-b}\left[\frac{\beta_\alpha}{\beta_{\alpha\alpha}}(b\kappa\sigma_1^{-1}+a\sigma_2^{-1}-b\kappa-a)+\alpha_\kappa\kappa ab(\sigma_2^{-1}-\sigma_1^{-1})\right], \tag{60}$$

$$\hat{\beta}/\hat{n}_2=\frac{1}{\tilde{\Delta}}\frac{n_2}{\beta}[\frac{\beta_\kappa\beta_\alpha}{\beta_{\alpha\alpha}}\kappa(1-\kappa)(1-\sigma_1^{-1})(1-\sigma_2^{-1})$$
$$+\frac{g}{1-b}\frac{\beta_\alpha^{\ 2}}{\beta_{\alpha\alpha}}(b\kappa\sigma_1^{-1}+a\sigma_2^{-1}-b\kappa-a)+\tilde{\beta}_\kappa\kappa\frac{ab}{1-b}(\sigma_2^{-1}-\sigma_1^{-1})g]<0. \tag{61}$$

Moreover, in the case of the population growth in unskilled labor equivalent to that in skilled labor, we have the following results.

$$\hat{a}/\hat{n}=\frac{1}{\tilde{\Delta}}n\{(1-\beta_\alpha)\frac{\beta_\alpha}{\beta_{\alpha\alpha}}+(\alpha_\kappa-\tilde{\beta}_\kappa)b\}\kappa(1-\sigma_2^{-1})(1-\sigma_1^{-1})>0, \tag{62}$$

$$\hat{b}/\hat{n}=\frac{1}{\tilde{\Delta}}n\{-(1-\beta_\alpha)\frac{\beta_\alpha}{\beta_{\alpha\alpha}}+(\alpha_\kappa-\tilde{\beta}_\kappa)b\}\kappa(1-\sigma_2^{-1})(1-\sigma_1^{-1}), \tag{63}$$

$$\hat{s}_L/\hat{n}=\frac{-1}{\tilde{\Delta}}n(1-\beta_\alpha)\frac{\beta_\alpha}{\beta_{\alpha\alpha}}\kappa(1-\sigma_2^{-1})(1-\sigma_1^{-1})>0, \tag{64}$$

$$\hat{a}/\hat{n}-\hat{b}/\hat{n}=\frac{-1}{\tilde{\Delta}}n(\alpha_\kappa-\tilde{\beta}_\kappa)(a+b)\kappa(1-\sigma_2^{-1})(1-\sigma_1^{-1})>0, \tag{65}$$

$$\hat{\alpha} / \hat{n} = \hat{\beta} / \hat{n} = \frac{1}{\tilde{\Delta}} n \frac{\beta_\alpha}{\beta_{\alpha\alpha}\alpha} \beta_\kappa \kappa (1-\kappa)(1-\sigma_2^{-1})(1-\sigma_1^{-1}) < 0 \cdot \tag{66}$$

Summarizing the outcomes, we have the following proposition.

Proposition 5

With endogenous factor-biased innovation that may shift with capital share in the three-factor case, with some relevant capital–skill complementarity technology as well as relatively large population growth in unskilled labor, the following holds in the steady state:

1. *Because of increasing capital share, contrary to the case where the innovation possibility frontier shifts with the capital accumulation, the increase in the saving rate and the advancement of capital-augmenting technology can move the innovation possibility frontier outward, and thus enhancing growth rate. In this case, the labor income inequality decreases although the aggregate labor income decreases.*
2. *However, because of decreasing capital share, the more increase in the population growth of unskilled labor can move the innovation possibility frontier inward, and thus may produce growth stagnate, produce more skill-biased innovation, and may increase labor income inequality.*
3. *If the population growth of unskilled labor is the same as that of skilled labor, contrary to the case where innovation frontier shifts with the capital accumulation, because of increasing capital share, population decline may move the innovation possibility frontier outward, and thus lead to having ambiguous effect on growth. However, the population decline is likely to lead to decreasing labor income inequality because the decrease in the skilled labor share is larger than that in the unskilled labor share.*

[Insert Figure 5]

Figures 5 shows the effect of population increase in the unskilled labor on the innovation possibility frontier. We note that the shift of innovation frontier plays a significant role for growth while the scarcity of skilled labor can provide the possibilities of enhancing skill-biased technology and labor income inequality in the induced innovation approach. Thus, if the shift of innovation frontier depends mainly on the capital share, the scarcity of skilled labor, implying the population growth in the unskilled labor can produce the growth stagnate, more skill-biased technology and more labor income inequality. Furthermore, in this case, capital-augmenting technology can enhance the frontier and thus promoting growth, but can lead to inequality, implying that the capital share increases and the aggregate labor share decreases.

Table 2. Effects of the parameters on labor shares, inequality, and biased innovation in some relevant capital-skill complementarity $\sigma_2 > 1 > \sigma_1$: The Case $\beta = \beta(\alpha, \kappa)$

	χ	$\tilde{c}$	a	b	s_L	a/b	κ	α	β	g
s	?	?	$-^a$	$-^b$	$-$	$-^a$	$+$	$+$	$+$	$+$
B	?	?	$-^a$	$-^b$	$-$	$-^a$	$+$	$+$	$+$	$+$
n_2	?	?	$+$	?	$+$	$+$	$-$	$+^c$	$-$	?
$n(=n_1=n_2)$	?	?	$+^d$	?	$+$	$+^d$	$-$	$-$	$-$	?

Note: $\tilde{\Delta} > 0$, $a < 0, d > 0$ if $\alpha_\kappa \geq \tilde{\beta}_\kappa$, $b < 0$ if $\alpha_\kappa \cong \tilde{\beta}_\kappa$, $c > 0$ if $\alpha_\kappa \cong 0$

These outcomes depend on the production structure of capital-skill complementarity because this capital-skill complementarity provides the direction of innovation and fluctuation of inequalities. Therefore, our outcomes may change if capital-augmenting technology such as artificial intelligence (AI) and automation can expand the frontier and moreover change the production structure into worker replacing structure such as capital-skill substitutability. Then, an alternative framework such as formulating endogenous capital-augmenting technical progress may be needed. [25]

4. Concluding remarks

Extending the innovation possibility frontier that includes skill-augmenting and unskilled-augmenting technical progress and the possible shift of the innovation frontier, we considered the determination of endogenous skill-biased innovation and the dynamics of income inequality with the heterogeneity of population growth of skilled and unskilled labor in the three-factor neoclassical growth model. We showed that the shift of the frontier can play significant role for growth and the relatively scarcity of skilled labor may produce more skill-biased innovation and labor income inequality in some relevant capital-skill complementarity. For example, in the case where the innovation frontier shifts with capital accumulation, if the population growth of unskilled labor is larger than that of skilled labor, this relatively scarcity of skilled labor supply may provide more skill-biased innovation, thus promoting growth, but produce the labor income inequality in the relevant capital-skill-complementarity. However, alternatively in the case where

[25] See Acemoglu and Restrepo (2017, 2018a, 2018b), Kotlikoff and Sachs (2012), Korinek and Stiglitz (2017), and Graetz and Michaels (2018). See also, Berg et al. (2018).

the innovation frontier shifts with capital share, the relatively scarcity of skilled labor supply may also provide more skill-biased innovation, but growth stagnate, and labor income inequality in some relevant capital-skill-complementarity.

However, to investigate the implications of automation and artificial intelligence technologies on growth, inequality, and unemployment, an alternative formulation of such a capital-augmenting technical progress and its analysis are needed. These issues will be addressed in future research.

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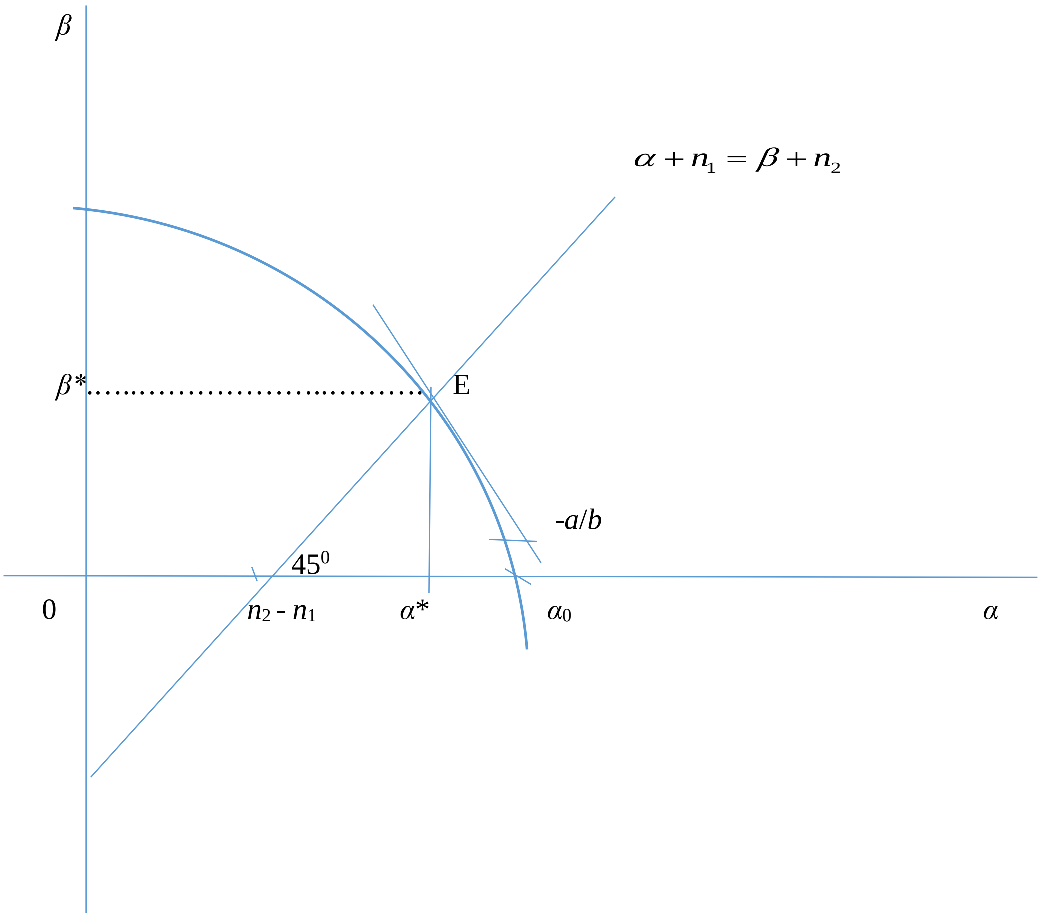

Figure 1 Equilibrium skill-biased innovation $\alpha^* > \beta^*$ when $n_2 > n_1$

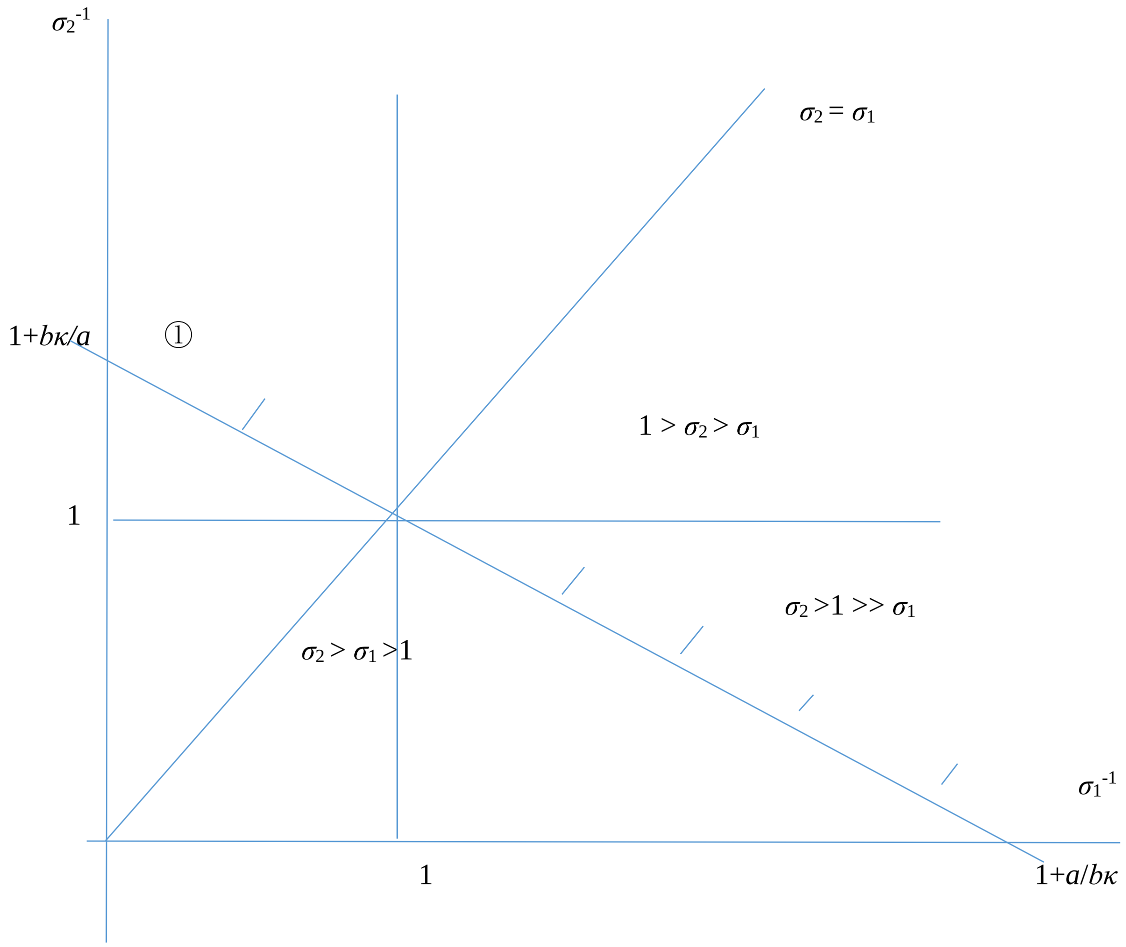

Figure 2 Stability condition in the case $\beta = \beta\,(\alpha,g)$: ① $a\sigma_2^{-1} + b\kappa\sigma_1^{-1} > a + b\kappa$

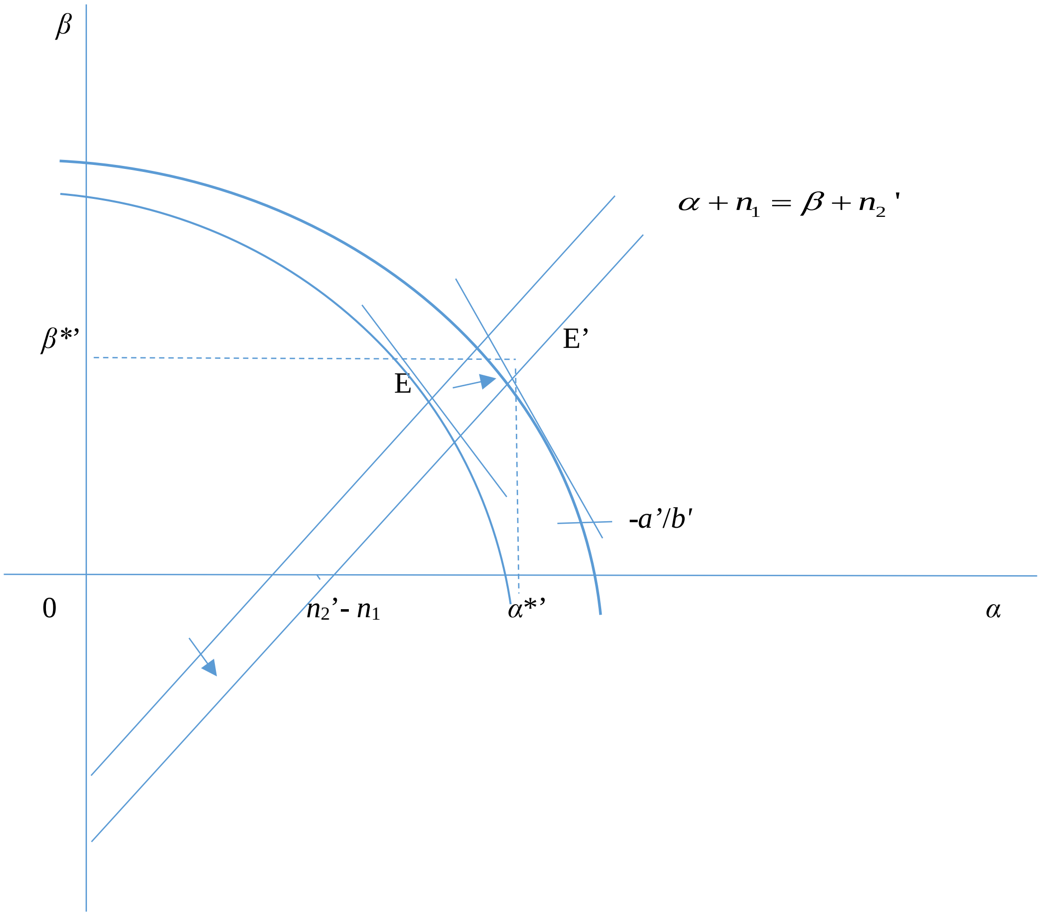

Figure 3 Effect of population increase in unskilled labor on the innovation possibility frontier in the case $\beta = \beta\ (\alpha, g)$

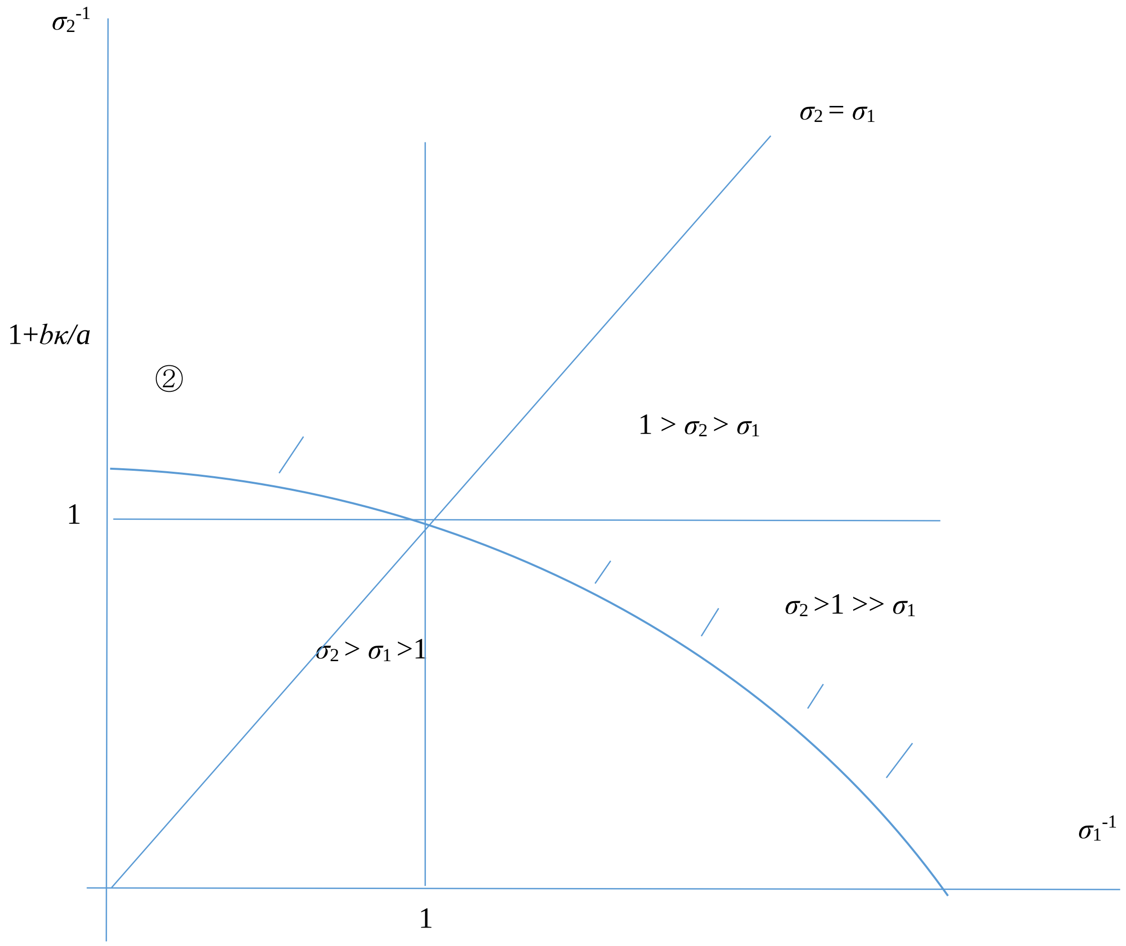

Figure 4 Stability condition in the case $\beta = \beta\,(\alpha,\kappa)$: ② $\tilde{\Delta} > 0$

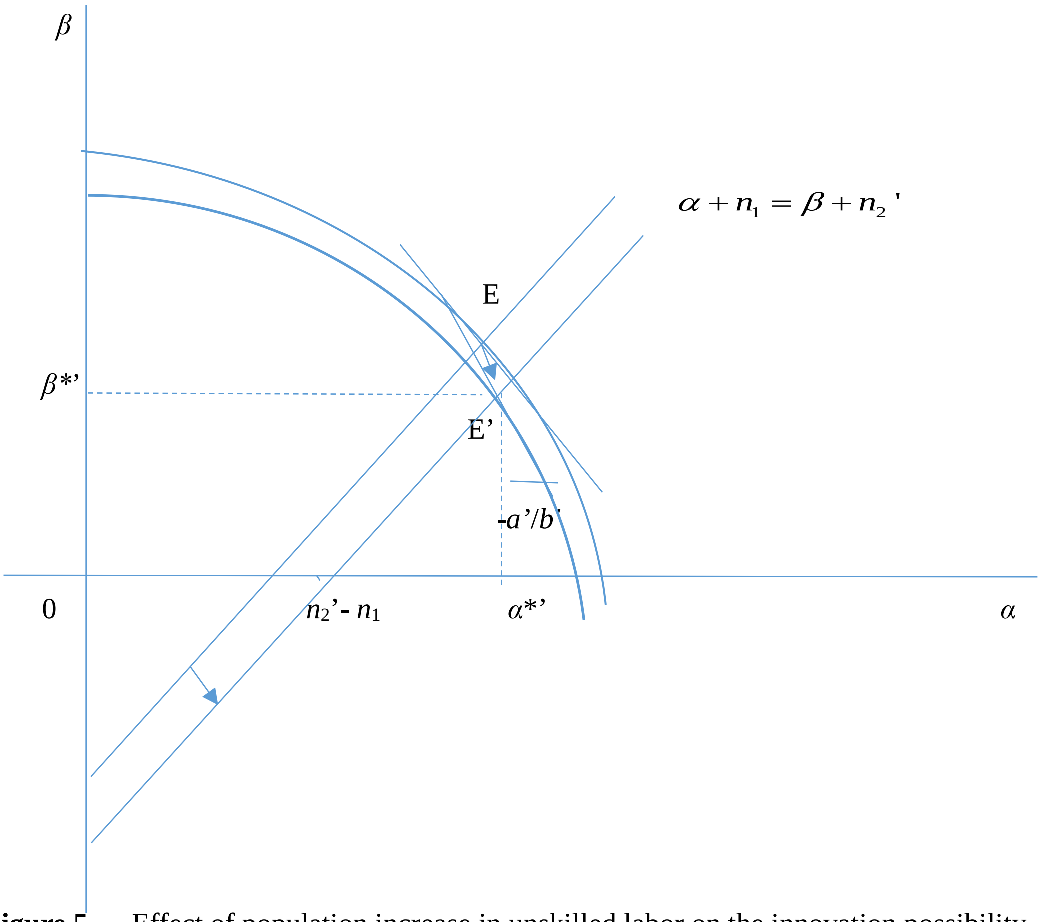

Figure 5 Effect of population increase in unskilled labor on the innovation possibility frontier in the case $\beta = \beta\,(\alpha, \kappa)$

Appendix 1

From equation (22), the results are given as as follows:

$$\hat{x}/\hat{s} = \frac{1}{\Delta}(1-\beta_\alpha-\beta_g)\frac{\beta_\alpha}{\beta_{\alpha\alpha}}\frac{1}{1-b}(-a\sigma_2^{-1}-\kappa\sigma_1^{-1}+a+\kappa)<0,$$

$$\hat{\tilde{c}}/\hat{s} = \frac{1}{\Delta}(1-\beta_\alpha-\beta_g)\frac{\beta_\alpha}{\beta_{\alpha\alpha}}\frac{-\kappa}{1-b}(\sigma_2^{-1}-\sigma_1^{-1})>0,$$

$$\hat{x}/\hat{B} = \frac{1}{\Delta}(1-\beta_\alpha-\beta_g)\frac{\beta_\alpha}{\beta_{\alpha\alpha}}\kappa(1-\sigma_1^{-1})<0,$$

$$\hat{\tilde{c}}/\hat{B} = \frac{1}{\Delta}(1-\beta_\alpha-\beta_g)\frac{\beta_\alpha}{\beta_{\alpha\alpha}}\frac{-\kappa}{1-b}(\sigma_2^{-1}-\sigma_1^{-1})>0,$$

$$\hat{x}/\hat{n}_2 = \frac{1}{\Delta}n_2\left[-\frac{\beta_\alpha}{\beta_{\alpha\alpha}}\frac{1}{1-b}(-a\sigma_2^{-1}-\kappa\sigma_1^{-1}+a+\kappa)+(1-\alpha_g)ag\right],$$

$$\hat{\tilde{c}}/\hat{n}_2 = \frac{1}{\Delta}n_2\left[\frac{\beta_\alpha}{\beta_{\alpha\alpha}}\frac{\kappa}{1-b}(\sigma_2^{-1}-\sigma_1^{-1})-(1-\alpha_g)(a+b)g\right],$$

$$\hat{x}/\hat{n}_1 = \frac{1}{\Delta}n_1\left[\frac{\beta_\alpha{}^2}{\beta_{\alpha\alpha}}\frac{1}{1-b}(-a\sigma_2^{-1}-\kappa\sigma_1^{-1}+a+\kappa)-(1-\tilde{\beta}_g)ag\right],$$

$$\hat{\tilde{c}}/\hat{n}_1 = \frac{1}{\Delta}n_1\left[-\frac{\beta_\alpha{}^2}{\beta_{\alpha\alpha}}\frac{\kappa}{1-b}(\sigma_2^{-1}-\sigma_1^{-1})+(1-\tilde{\beta}_g)(a+b)g\right]$$