The Falling Labor Share and the Autor and Dorn model*

Shiro KUWAHARA†

May 8, 2020

Abstract

This paper surveys the empirical studies that report change, especially decrease, of labor share in the modern economies, and then picks up the the Autor and Dorn (2013) model, which is able to depict the change of labor allocation between goods production and service sector. We investigate the working of the model and derive some directions of extensions to replicate the fall of labor share, among others.

1 Introduction

Although many empirical studies have been showing the stable labor share in the post World War II (for example, Maddison 1982), we have recent empirical studies that imply the declining labor share such as Blanchard (1997), Karabarbounis and Neiman (2014), Autor et al. (2017), Dao et al.(2017), which report this phenomena is not limited by the advanced economies but also caught by developing countries. This polarization seem to have been breaking down the abundant middle class that insists of industrial workers, and they are the core member of the Affluent Society (Galbraith 1958). This process already yields some poverties and various social problems.

Thus, the polarization of the world *à la* 1970s, the co-existence of growing advanced economies and economies caught by poverties, was surmounted by the starts of substantial economic growths by some developing countries (see, for example, World Bank 1993), but nowadays, another polarization of the world *à la* 21th century seem to be running on, which is the reemergence of

*This paper greatly owe to the discussion with Tatsuichi Kaneko. Of course, all remaining errors are sole responsibility of the author.

†corresponding author, University of Hyogo, E-mail: kuwahara@em.u-hyogo.ac.jp

class society that has been resolved as the fruits of these several decades of powerful and global economic growth.

On this process of world economy's transition, some scholars recognize the change, for example, D. Bell (1974)'s post- industrialization, A. Toffler(1980)'s "the third wave." These are ones that point out the expiry of the era of industrial production, of course, the production itself does not cease, but the main engine of the economy is change into knowledge creation. This is considered as emerging the world with knowledge-based growth. This is also called as the information - oriented society, and it is especially characterized by the persistent innovation on computer industry, for example, a famouse Moore's law. This is not an only change, another important change is the tertiary industrialization, which is famous as Petty-Clark's law (Petty 1690, Clark 1940).

Thus, this paper pick up the Atour and Dorn (2013) model, which contains the persistent innovation on computer industry and dynamic allocation of unskilled labor between service and goods prodcution. On this process, one of key concept seems to be erasticity of substitution between capital and labor. Karabarbounis and Neiman (2014) estimate an elasticity of substitution greater than 1, and conclude that decreasing relative prices of capital goods urge the shift of input factor from labor to capital. But, for example, Chirinko (2008), for example, who surveyed and evaluated a large number of studies that attempted to measure this elasticity, concluded that a value of the elasticity of substitution is suggested in the range of 0.4 to 0.6. [1] Roughly speaking, the simple way to relate the decreasing price of capital and the decreasing labor share is the elasticity of substitution greater than 1, but the many empirical studies imply that that is lower than 1.

Of course, we have the newer study that obtains the one greater than 1: Using micro panel data from the U.S. Economic Census since 1982, Autor et al.(2019) demonstrate empirical patterns to assess a new interpretation of the fall in the labor share based on the rise of " superstar firms. " If globalization or technological changes push sales towards the most productive firms in each industry, product market concentration will rise as industries become increasingly dominated by superstar firms, which have high markups and a low labor share of value-added.

Schwellnus et al.(2018) makes the following assignments: Technological change in the investment goods-producing sector and greater global value

[1] We can refer many studies such as Antras (2004), Chirinko, Fazzari, and Meyer (2011), Oberfield and Raval (2014), Chirinko and Mallick (2014), Herrendorf, Chirinko, Fazzari, and Meyer (2011), Herrington, and Valentinyi (2013), Oberfield and Raval (2014), Chirinko and Mallick (2014), Herrendorf, Herrington, and Valentinyi (2013), and Lawrence (2015). They all estimate elasticities below 1.

chain participation have declined labor shares, but the effect of technological change has been significantly less clear. Countries with falling labor shares have experienced both a decline at the technological frontier and takeover of market shares by top advanced firm with low labor shares.

In this paper, we focus on the decreasing labor share by using a model with the elasticity of substitution. In this literature, many interesting models have been proposed. For example, using general form of the neoclassical storucture, which contains multisector tied with constant elasticity of substitution (CES) technology, Accemoglu (2003) shows that along the transition path, there is capital-augmenting technological change and factor shares change, but in the long run, economy endogenously becomes the labor-augmenting one, which is described by standard growth models. This paper provides microfoundations where an economic agent endogenously choose labor-augment technological change, and imples that any distortion or disturbance such as tax policy and changes in labor supply or savings might change factor shares in the short run, but have no, or at least little, effect on the long-run factor distribution of income. The necessary of the labor-augment innovation for long-run growth itself is a well-known as (a part of) the Uzawa theorem (Uzawa 1961). However, this condition contradicts with the obserbed steadily falling relative price of capital equipment adjusted quality.

Thus, it seems that standard model is not sufficient to inquire the social probrem that conatins polarization into labor and capitaist class, and a model that contains further detail labor storucture should be needed.

Here, we pick up the Autor and Dorn (2013) model, where unskilled labor is divided into routine (emploed in good production) and manual (did in service sector), andthe tansition of labor force from good procution into service sector is descrived. The model is intereseting and successes some aspects of real phenomena, but since theoretical interest is not a sole objenction of the ariticle, some results are not explicitly shown and some important factors are eliminated. Thus, we sketck the model and shows the further directions of the reserch of the extensions of the Autor and Dorn (2013).

The organization of this paper is as follows. Section 2 give a description of the model. Section 3 derives the asymptotic steady states in the long run. Section 4 descrive the market economy of the model and drive some properties of it. Section 5 discuss the necessities of futurer extensions. And at the last, Section 6 concludes the paper.

2 The description of the model

2.1 Final goods production and Labor Supply

Following Autor and Dorn (2013), we assume that the final goods (Y_g) are produced by abstract work (L_a) that is executed by employing skilled labor, and composited intermediate input denoted Z, and Z is made by routine work (L_r) by employing un-skilled labor and computure (X). We also assume that the erastisity of substitution between L_a and Z on Y_g production is 1, and that between L_r and X on contutitution of Z is $\sigma_r > 1$. Thus, we obtain the following function form:

$$Y_g = L_a^{1-\beta}\left\{(\alpha_r L_r)^\mu + (\alpha_x X)^\mu\right\}^{\frac{\beta}{\mu}}, \quad \beta, \mu \in (0,1), \tag{1}$$

where Z defined as

$$Z \equiv \left\{(\alpha_r L_r)^\mu + (\alpha_X X)^\mu\right\}^{\frac{1}{\mu}}, \quad \mu > 0$$

where it should be noted that $\mu > 0$ implies $\sigma_r = \frac{1}{1-\mu} > 1$, which captures the property that computer and routine labot are 'gross substitutes'. This arrangement immediately means that routine labor is substituted by computer for cheaper conputer price (denoted by p_x).

The low-skill workers supply either *manual* or *routine* labor. The low-skill labor (we denote L_U) is inerastically supplied at unit mass ($L_U = 1$).

The skill of the manural labor is homogenous. So if all worker are employed in the manual labor, the labor supply for manual work (L_m) is unity ($L_m = L_U = 1$).

In the routine work, there is the efficiency denoted η with the density function $f(\eta)$, and each worker has her inherent efficiency. We assume that the aggregate efficiency labor supply for routine work is also unity, so we have

$$\int_{\eta \in U} \eta f(\eta) d\eta = 1 \tag{2}$$

where S denote the set of unskilled labor, and the labor supply of routine labor is given as which is defined as

$$L_r = \int_{\eta \in U_r} \eta f(\eta) d\eta \tag{3}$$

where U_r denotes the set of labor who work as routine worker.

It should be noted that L_a is the number of employee in the final goods sector, but since the routine work contains efficiency parameter, L_r denotes the efficiency number of employee.

For the convenience of obtaining an analitical solution, we specify the density function. Following Autor and Dorn (2013), we also adopt the exponential distribution:

$$f(\eta) = e^{-\eta}, \quad \text{for } \eta \in [0, \infty]$$

We reapectively denote wage rate offered for unit of manual and routine labor as w_m and w_r. Each worker with efficiency η can gain w_m if s/he works as manual worker, and $w_r\eta$ if does as routine worker.

Each worker select the work only depending on labor revenue, and wage rates are given, the worker with $\eta \geq w_m/w_r (\equiv \bar{\eta})$ works as routine worker, and the worker with $\eta < \bar{\eta}$ works as manual worker. Thus, $\bar{\eta}$ represent the threshold efficiency that devides the type of work which the unskilled-worker select. Thus, (3) is rewritten as

$$L_r = \int_{\bar{\eta}}^{\infty} \eta f(\eta) d\eta. \tag{4}$$

Then, we derive the resource constraint of labor.

$$1 = L_m + \int_{\bar{\eta}}^{\infty} f(\eta) d\eta \tag{5}$$

This equation gives the following equation (see Appedix for detail derivation):

$$L_r = \big\{1 - \log(1 - L_m)\big\}(1 - L_m)(\equiv g(L_m)). \tag{6}$$

2.2 Conditions derived from optimaizations

We here obtain some conditions derived from three prodcution sectors, final goods, service, and intermediate goods, and household. All production sectors are assumed to be perfectly competitive. So each sector's price equates to its marginal cost, and no profit is left on firm.

The Final Goods Sector The profit of the final goods sector π_g is given as

$$\pi_g = Y_g - w_a L_a - w_r L_r - p_x X,$$

where we adopt the final goods as a numéraire, so the price of the final goods is unity.

The optimal conditions derived by final goods firm are given as follows:

$$\frac{\partial \pi_g}{\partial L_a} = 0 \Longrightarrow \frac{\partial Y_g}{\partial L_a} = w_a \tag{7}$$

$$\frac{\partial \pi_g}{\partial L_m} = 0 \Longrightarrow \frac{\partial Y_g}{\partial L_r} = w_r \tag{8}$$

$$\frac{\partial \pi_g}{\partial X} = 0 \Longrightarrow \frac{\partial Y_g}{\partial X} = p_x \tag{9}$$

The Service Sector The profit of service sector π_s is given as

$$\pi_s = p_s Y_s - w_m L_m$$

and the production technology is given as

$$Y_s (= C_s) = \alpha_s L_m \tag{10}$$

Constant returns of this priduction technology yields the zero-profit condtion (instead of F.O.C.) as follows:

$$(p_s Y_s =) p_s C_s = w_m L_m \tag{11}$$

and uniting (10) and (11), we have

$$p_s \alpha_s = w_m. \tag{12}$$

This condition implies that the price of service p_s and the wage of manual labor w_m are tied.

The Intermediate goods (computer) sector The profit of intermediate goods sector π_x is given as

$$\pi_x = p_x X - Y_x$$

and the production technology is given as

$$X = \delta_0 Y_x e^{\delta t}, \tag{13}$$

where a positive constant δ denotes exogenously-given innovation rate of intermediate goods that is presumed as computer. δ_0 (θ in Autor and Dorn 2013) denotes the efficiency parameter, which equals the price of X at initial period $t = 0$ (in Autor and Dorn 2013, the initial period is $t = 1$). Followinig Autor and Dorn (2013), we set the price of X at initial period to unity, which yield $\bar{\delta} = 1$.

Since this sector is also constant returns to scale, obtained condtion is the zero-profit condition:

$$p_x = \frac{Y_x}{X} = e^{-\delta}. \tag{14}$$

Household To close the model, we assume that all consumers/workers have identical CES utility functions over consuming final goods and servise:

$$u = (c_g^{\rho} + c_s^{\rho})^{\frac{1}{\rho}}, \quad \rho < 1. \tag{15}$$

The elasticity of substitution in consumption between goods and service is derived as $\sigma = \frac{1}{1-\rho}$.

3 The long-run steady states

Since the model contains no distortion (monopoly, externality, and so on), we can derive the equilibrium conditions by considering a social planner problem, and it is convenient way to get the long-run allocation between L_m and L_r. Normalized population scale makes the identification of per capita and aggregate values possible.

The Planner problem Since the resource constraint of final goods $Y_g = C_g + Y_x$ and $Y_x = p_x X$ from (14), we have $C_g = Y_g - p_x X$. Thus, given the sequence of p_x, the social planner problem is given as

$$\max_{X, L_m} \left[L_m^{\frac{\sigma-1}{\sigma}} + \left(Y_g - p_x X\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{16}$$

which is subject to (1) and (6). Thus, the basic structure of the Autor and Dorn (2013) model is the continuous sequence of instantaneous decision making, and unique intertemporal factor is the incessantly-decreasing factor price of computer goods. This captures the innovation in this study.

This problem yields the following two conditions:

$$\frac{\partial Y_g}{\partial X} = p_x, \tag{17}$$

$$L_m^{-\frac{1}{\sigma}} = (Y - p_x X)^{-\frac{1}{\sigma}} \frac{\partial Y_g}{\partial Z} \frac{\partial Z}{\partial L_r} \left\{ -\log(1 - L_m) \right\},. \tag{18}$$

From the discussion given in Appendix A, we obtain the following condition from (18):

$$\frac{1}{\sigma} \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \frac{\beta - \mu}{\beta} \Rightarrow \lim_{X \to \infty} X^{\beta - \mu - \frac{\beta}{\sigma}} = \left\{ \begin{array}{c} \infty \\ 1 \\ 0 \end{array} \right. \Rightarrow L_m = \left\{ \begin{array}{l} 0 \\ L_m^* \in (0,1) \\ 1 \end{array} \right. . \tag{19}$$

Asymptotic Labor Allocation Since price of computer goods p_x falls to zero asymptotically, (17) implies that computer goods limits to

$$\lim_{t\to\infty} X(t) = \infty.$$

From this and $L_r \leq 1 << \infty$, we obtain

$$\lim_{t\to\infty} \frac{Z}{\alpha_x X} = 1.$$

Following Autor and Dorn (2013), let $x \sim y$ be a shorthand for the notation that $\lim_{t\to\infty} x/y = 1$, we can obatain following expressions:

$$Y_g = \underbrace{L_a^{1-\beta}}_{=1} Z^\beta \sim (\alpha_x X)^\beta \tag{20}$$

$$p_x = \frac{\partial Y_g}{\partial X} \sim \beta \alpha_x^\beta X^{\beta-1} \tag{21}$$

Therefore, $p_x X \sim \beta(\alpha_x X)^\beta$. Then, substituting equations obtained above, $Y_g \sim (\alpha_x X)^\beta$ and $p_x X \sim \beta(\alpha_x X)^\beta$, and the zero profit condition of computer sector, $p_x X = Y_x$, into the resource constraint of goods ($Y_g = C_g + Y_x$), we have

$$C_g = Y_g - p_x X \sim (\alpha_x X)^\beta - \beta(\alpha_x X)^\beta = \kappa_1 X^\beta = (1-\beta) Y_g, \tag{22}$$

where $\kappa_1 \equiv (1-\beta)\alpha_x^\beta$.

From the resource constrain of final goods, we have $Y_g = Y_x + C_g$, then

$$Y_g = Y_x + C_g \sim X e^{-\delta t} + \kappa_1 X^\beta = (\alpha_x X)^\beta \tag{23}$$

Therefore, we obatin $Y_X = X e^{-\delta t} = (\alpha_X^\beta - \kappa_1) X^\beta = \beta \alpha_x^\beta X^\beta$, which yields

$$\frac{\dot{Y}_X}{Y_X} = \frac{\dot{X}}{X} - \delta = \beta \frac{\dot{X}}{X}, \tag{24}$$

which yields

$$\frac{\dot{X}}{X} = \frac{\delta}{1-\beta}. \tag{25}$$

From (23) and (25), we have

$$\frac{\dot{Y}_g}{Y_g} = \frac{\dot{Y}_x}{Y_x} = \frac{\dot{C}_g}{C_g} = \beta \frac{\dot{X}}{X} = \frac{\beta\,\delta}{1-\beta}. \tag{26}$$

Thus, we obtain the following lemma:

Lemma 1 We have following results that

$$\lim_{t\to\infty} p_x X = \infty, \quad \lim_{t\to\infty} \frac{p_x X}{Y} = (const) \in (0,\infty).$$

Proof) We obtain $\frac{\dot{p}_x}{p_x} = -\delta$ (from (14)), and $\frac{\dot{X}}{X} = \frac{\delta}{1-\beta} > \delta$, which yield $\frac{\dot{(p_x X)}}{p_x X} = -\frac{\delta\beta}{1-\beta} > 0$. Furthermore, uniting $\frac{\dot{Y}_g}{Y_g} = \frac{\delta\beta}{1-\beta}$, $\frac{p_x X}{Y}$ is a constant positive value less than infinity. (Q.E.D)

4 Description of the market economy of the Autor-Dorn model

Here, we explore the relationship between goods and service. Autor and Dorn (2017) analyze the model by solving the social planner probrem, so the price of service does not emerge, therefore, GDP, sum of market values of good and service product can not be calculated. In this survey, we analyze the working of the market economy, which makes the derivation of GDP possible. Thus, we introduce the price dynamics of service (p_s). Under perfect competition, we have zero profit condition of service sector (which yields $p_s\alpha_s = w_m$), arbitrage condition between routine and manual labor (which yields $w_m = w_r$), and marginal principle of routine work (which yields $w_r = \frac{\partial Y_g}{\partial L_r}$), therefore, we obtain the following equation:

$$p_s\alpha_s = w_m = w_r = \frac{\partial Y_g}{\partial L_r} \tag{27}$$

where $w_m = w_r$ holds under $L_m \in (0,1)$. The utility function we use in this study is given in (15), and this economic agent is a representative household, and therefore, the family structure is an epitome of the whole economy's population structure, so this household have the labor endowment with a distribution function given $f(\eta) = e^{-\eta}$, which we assume the distribution of labor efficiency. Since each labor endowment is assumed to be normalized to 1, the aggregate, average and representative household's values in this study can be denoted as the same. Thus, the maximizing problem of the representative household can be given as follows:

$$\max_{c_g,c_s} \quad \left(C_g^\rho + C_s^\rho\right)^{\frac{1}{\rho}}, \tag{28}$$

$$\text{s.t.} \quad C_g + p_s C_s = w_m L_m + w_r L_r + w_a L_a. \tag{29}$$

ρ	$-\infty$	$\cdots$	0	$\cdots$	1
σ_u	0	$\cdots$	1	$\cdots$	∞
$p_s \sim$			∞		

Table 1: Long-run value of p_s

This problem yields the following optimal condition:

$$\left(\frac{C_s}{C_g}\right)^{\rho-1} = p_s \tag{30}$$

Then, using $X \sim \infty$, we have

$$\begin{aligned} w_r L_r &= \frac{\partial Y_g}{\partial L_r} L_r = \frac{\beta}{\mu} L_r^{1-\beta} [\ldots]^{\frac{\beta}{\mu}-1} \mu \alpha^\mu L_r^{\mu-1} L_r \\ &= \beta \frac{\alpha_r^\mu L_r^\mu}{\alpha_r^\mu L_r^\mu + \alpha_x^\mu X^\mu} Y_g \sim (a_x X)^{\beta-\mu}. \end{aligned} \tag{31}$$

This yields $\frac{w_r L_r}{Y_g} = (\alpha_x X)^{-\mu} \sim 0$, which implies routine share in goods production converges to 0 (infinitely small).

From (22), we have the following equation:

$$C_g = (1-\beta) Y_g. \tag{32}$$

This implies that in the long run, the consumption C_g is converging to the constant rate of Y_g, consumption propensity (C_g/Y_g) depends on β, and not on σ and μ, and then goods consumption (C_g) is growing infinitely. Whereas, the product, and namely consumption, of servie is given by $(Y_s =)C_s = \alpha_m L_m$, and constant long-run value of L_m, which is becase (unskilled) labor endowment is assumed to be constant, yields constant long-run value of $C_s (= Y_s)$. Therefore, $p_s \sim \infty$ is realized for $\forall \rho \in (\infty, 1)$, given in Tab.1. Incessant innovation of computer makes incessant price down of computer and it realizes the incessant goods' price down. In this study, goods' price is taken as numéraire, the incessant goods' price down is represented by incessant service price up. This price change disturbs the balance of share between goods and service, and this imbalance is adjusted through the shift of unskilled labor between routine and manual labor.

GDP Introduction of prices makes us to analyze the whole economic activity lebel, that is, of course, GDP. In this case, it is natural to define GDP

ρ	$-\infty$	$\cdots$	0	$\cdots$	1
σ_u	0	$\cdots$	1	$\cdots$	∞
	$\frac{\dot{p}_s}{p_s} > \frac{\dot{Y}_g}{Y_g}$		|	$\frac{\dot{p}_s}{p_s} < \frac{\dot{Y}_g}{Y_g}$	
$Y \sim$	$p_s Y_s$		|	Y_g	
$\dot{Y}_g/Y_g \sim$	$\frac{(1-\rho)\beta\delta}{1-\beta}$		|	$\frac{\beta\delta}{1-\beta}$	

Table 2: Long-run value of Y_g and $\dot{Y}_g/Y_g$

as $Y \equiv Y_g + p_s Y_s$, since these two are the whole production in this economoy. From (11), (30) and (32), we have

$$p_s = \left(\frac{(1-\beta)Y_g}{\alpha_m L_m} \right)^{1-\rho}. \tag{33}$$

When $L_m \neq 0$ holds, namely, under $\forall \rho < 0$ and $\exists \rho \in (0,1)$, the growth of L_m stops in the long run, therefore, (30) and (32) yields

$$\frac{\dot{p}_s}{p_s} = (1-\rho)\frac{\dot{Y}_g}{Y_g}.$$

Uniting the above equation and (26), we have the following condition:

$$\frac{\dot{p}_s}{p_s} \sim \frac{\beta\delta}{1-\beta}(1-\rho) \left\{ \begin{array}{c} > \\ < \end{array} \right\} \frac{\dot{Y}_g}{Y_g}, \quad \text{for} \quad \rho \left\{ \begin{array}{c} \leq 0 \\ \in (0,1) \end{array} \right. . \tag{34}$$

This condition immediately gives

$$Y \sim \left\{ \begin{array}{c} p_s Y_s \\ Y_g \end{array} \right. , \quad \text{for} \quad \rho \left\{ \begin{array}{c} \leq 0 \\ \in (0,1) \end{array} \right. . \tag{35}$$

For the case of $\rho < 0$, namely, the case of complimentary, the long-run main sector, which means a sector produce added value most, becomes the service sector. Thus, ρ, namely, elasticity of substitution on consumption $\sigma_u = 1/(1-\rho)$ determines the main (most valued) sector. Furthermore, the growth rate of service-oriented economy is higher than that of goods-production economy.

Labor allocation and Distribution Next, we analyze the labor allocation and distribution on labor.

Thereshold on more valued production whether service or good is $\rho = 0$ namely $\sigma = 1$, but threshold on labor allocation between manual and routine is $1/\bar{\sigma} = \frac{\beta-\mu}{\beta}(< 1)$, which is placed on $\bar{\sigma}(> \sigma)$ (See Appendix 3.3).

ρ	$-\infty$	$\cdots$	0	$\cdots\cdots$	1
σ_u	0	$\cdots$	1	$\cdots\lvert\bar{\sigma}\rvert\cdots$	∞
$L_m \sim$		1		$\lvert L_m^*\rvert$	0
$Y \sim$		$p_s Y_s$	$\vert$	Y_g	

Table 3: Long-run value of Y_s under $\beta > \mu$

ρ	$-\infty$	$\cdots$	0	$\cdots\cdots$	1
σ_u	0	$\cdots$	1	$\cdots$	∞
$L_m \sim$			1		
$Y_s \sim$		$p_s Y_s$	$\vert$	Y_g	

Table 4: Long-run value of Y_s under $\beta < \mu$

In the case of $\beta > \mu$, $\bar{\sigma} > 1$, thus, we have the following values: In the case of $\beta < \mu$, $\bar{\sigma} < 0$, thus, we have the following values:

Thus, only sufficiently low μ could yields the asymptotic steady state with $L_m \sim 0$, and uniting this and sufficiently large σ actually yiedls that steady state.

However, the globally properties depends on ρ, and the economy with $\rho < 0$ yields serive-weighted economy and the one with $\rho > 0$ does goods-weighted economy. It should be noted that this is not derived from the utility, but it derived from production structure, where good production diverses but service production has upper bound caused by the given production factor endowment.

On the analysis of Autor and Dorn (2013), since skilled labor work in the goods sector and their income is assumed to be maintain, they assume $\rho > 0$, and as a result, GDP, aggregare added value, mainly depend on the goods production, so this arrangement can not depict the "tertiary industrialization". Furthermore, when we use the word "tertiary industrialization", we would consider that the service sector becomes the main sector of the economy, and in the real world, the most davanced sectors are financial sector or ITC sector that provide service, but the service sector of the present model is, at most, retail business that mainly employs the unskilled labor for sales people. Introducing an advanced service sector, such as financial sector, would be necessary to cover the tertiary industrialization for the important thema.

5 Concluding remark

From the above discussions, we conform the following features of the Autor and Dorn (2013) model: (i) long-run phase is determed by the utility parameter, and under $\rho > 1$, long-run main engine is fixed at the goods production sector. (ii) Under the assumption of (i), the long-run labor allocation is determined by the goods productive. (iii) The model essentially lacks the intertemporal decision making, which makes the meaning of interest rate, therefore, that of capital share, ambigurous. From these properties, we can have the direction of future reserches that is extended from the Autor and Dorn (2013) model.

Furthermore, for the purpose of the analyzing the transition of the labor share, the dynamics of the labor supply, as well as capital accumulation, endogenizing labor supply would be effective.

Of course, final and ultimate goal of the studies are increasing the welfare of the world, and from the point of this view, many economic agent is a labor as well as a citizen, the resolution of the decreasing labor share, which implicitly yields the decreasing labor income and afford for social lives in broad meaning, just as Keynes (1930) makes a perspecitve thatlabor is shorter in the future.

6 Appendix

6.1 The derivation of the function $g(\cdot)$

From the resource constraint of labor, we have

$$1 = L_m + \int_{\bar{\eta}}^{\infty} f(\eta) d\eta. \tag{36}$$

Here, L_r denotes the effective labor input on the routine sector, namely efficency η times the density of labor with efficiency η, $f(\eta)(= e^{-\eta})$, therefore, we have

$$L_r = \int_{\bar{\eta}}^{\infty} \eta e^{-\eta} d\eta.$$

Caliculating this by using rule, we have

$$\begin{aligned}
L_r &= \int_{\bar{\eta}}^{\infty} \eta e^{-\eta} d\eta = \left[-\eta e^{-\eta} - e^{-\eta}\right]_{\bar{\eta}}^{\infty} \\
&= \bar{\eta} e^{-\bar{\eta}} + e^{-\bar{\eta}} - \underbrace{\lim_{\eta\to\infty} \bar{\eta} e^{-\bar{\eta}}}_{\to 0} - \underbrace{\lim_{\eta\to\infty} e^{-\bar{\eta}}}_{\to 0} \\
&= (1+\bar{\eta}) e^{-\bar{\eta}}. \qquad (37)
\end{aligned}$$

Then, we concider the resource constraint on the unskilled labor. From (5), we have

$$\begin{aligned}
L_m &= 1 - \int_{\bar{\eta}}^{\infty} e^{-\eta} d\eta \\
&= 1 - \left[-e^{-\eta}\right]_{\bar{\eta}}^{\infty} = 1 - [-\underbrace{\eta^{-\infty}}_{\to 0} + e^{-\bar{\eta}}] \\
&= 1 - e^{-\bar{\eta}}. \qquad (38)
\end{aligned}$$

What we want is the relationship between L_r and L_m, so we eliminate $\bar{\eta}$. For this purpose, we derive the value of $e^{-\bar{\eta}}$ from (37), and so did that of $\bar{\eta}$ from (38):

$$\log(1 - L_m) = \log e^{-\bar{\eta}} = -\bar{\eta}. \qquad (39)$$

From (38) and (39), we respectively obtain $e^{-\bar{\eta}} = 1 - L_m$, and $\bar{\eta} = -\log(1 - L_m)$, and substituting these two equations into (37) for eliminating $\bar{\eta}$, we get the function $g(\cdot)$ as follows:

$$L_r = \left\{1 - \log(1 - L_m)\right\}(1 - L_m)(\equiv g(L_m)). \qquad (40)$$

6.2 Derivation of Eq.(19) from Eq.(18)

In the planner probrem, we obtain the following conditions:

$$\frac{\partial U}{\partial X} = 0 \Rightarrow \frac{\partial Y_g}{\partial X} = \beta \frac{Y_g}{Z} \alpha_x^{\mu} X^{\mu-1} = p_x (= e^{-\delta t}), \qquad (41)$$

$$\frac{\partial U}{\partial L_m} = 0 \Rightarrow L_m^{-\frac{1}{\sigma}} = (Y_g - p_x X)^{-\frac{1}{\sigma}} \frac{\partial Y_g}{\partial L_m}, \qquad (42)$$

where we have the following:

$$\frac{\partial Y_g}{\partial L_m} = \frac{\partial Y_g}{\partial Z} \frac{\partial Z}{\partial L_r} g'(L_m) = -\beta L_a^{1-\beta} Z^{\beta-\mu} \alpha^{\mu} L_r^{\mu} \frac{dg(L_m)}{dL_m}. \qquad (43)$$

The partial feivatives are made into

$$\frac{\partial Y_g}{\partial Z} = \beta Z^{\beta-1} \tag{44}$$

$$\frac{\partial Z}{\partial L_r} = Z^{1-\mu}\alpha^\mu L_r^\mu, \tag{45}$$

Here, we have

$$L_a = 1, \tag{46}$$

$$g'(L_m) = \log(1-L_m) = -\bar{\eta}, \tag{47}$$

$$\int_{\bar{\eta}}^{\infty} e^{-\eta} d\eta = \left[-e^{-\eta}\right]_{\bar{\eta}}^{\infty} = 0 - (-e^{-\bar{\eta}}) = e^{\bar{\eta}}. \tag{48}$$

From (47), we obtain

$$e^{-\bar{\eta}} = 1 - L_m, \quad \text{namely} \quad L_m + e^{-\bar{\eta}} = 1, \tag{49}$$

which is just a resource constraint on unskilled labor.

Here, we impose the conditions on asymptonic staedy states as follows:

$$C_g \sim \kappa_1 X^\beta, \tag{50}$$

$$Z \sim \alpha^\mu X^\mu. \tag{51}$$

Substituting above conditions (44), (45) and (46) into (43), we have

$$L_m^{-\frac{1}{\sigma}} = -\kappa_L (X^\beta)^{-\frac{1}{\sigma}} L_a^{1-\mu} Z^{\beta-\mu} g(L_m)^{\mu-1} g'(L_m) \tag{52}$$

where $\kappa_L \equiv \kappa_1^{-\frac{1}{\sigma}} \alpha_r^\mu \beta$, and $g'(L_m) = \log(1-L_m)(= -\bar{\eta})$.

Thus,

$$\lim_{t\to\infty} \frac{\partial U}{\partial L_m} = 0 \quad \sim \quad \underbrace{L_m^{-\frac{1}{\sigma}} g(L_m)^{1-\mu} g'(L_m)^{-1}}_{\equiv \Lambda(L_m)} = \lim_{X\to\infty} \kappa_L X^{\beta-\mu-\frac{\beta}{\sigma}} \tag{53}$$

Therefore, the limit value of the RHS depends on the exponential parameter $\beta - \mu - \frac{\beta}{\sigma}$. The results are summarized as follows:

$$\beta - \mu - \frac{\beta}{\sigma} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \Leftrightarrow \frac{1}{\sigma} \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \frac{\beta-\mu}{\beta} \Rightarrow \lim_{X\to\infty} X^{\beta-\mu-\frac{\beta}{\sigma}} = \left\{ \begin{array}{c} \infty \\ 1 \\ 0 \end{array} \right. . \tag{54}$$

This condition is derived from the optimal conditions derived from the planner probrem, so L_m must be controlled to satisfy this equation. In desentralized economy, it is also satisfied by the optimizing behavior of agents.

To derive the behavior of L_m, it is necessary toconfirming the properties of $\Lambda(\cdot)$. Taking the limit of the function Λ, we have the followings:

$$\lim_{L_m \to 0} \Lambda(L_m) = \underbrace{0^{-\frac{1}{\sigma}}}_{\to\infty} \underbrace{\left[\{1 - \underbrace{\log(1-0)}_{\to \log 1 = 0}\}(1-0)\right]^{1-\mu}}_{\to 1} \underbrace{\frac{1}{\log(1-0)}}_{\to\infty} = \infty,$$

$$\lim_{L_m \to 1} \Lambda(L_m) = \underbrace{1^{-\frac{1}{\sigma}}}_{\to 1} \underbrace{\left[\{1 * (1-1) - (1-1)\log(1-1)\}\right]^{1-\mu}}_{\to 0} \underbrace{\frac{1}{\log(1-1)}}_{\to 0} = 0.$$

On the middle item of the RHS, we have

$$\lim_{L_m \to 1} [\{1 - \log(1-L_m)\}(1-L_m)] = \lim_{\mathcal{L} \to 0} \mathcal{L} - \mathcal{L}\log\mathcal{L} = 0 - 0 = 0, \quad (55)$$

where we use the following result: $\lim_{\mathcal{L}\to 0} \mathcal{L}\log\mathcal{L} = \lim_{\mathcal{L}\to 0} \frac{\log\mathcal{L}}{1/\mathcal{L}} = \lim_{\mathcal{L}\to 0} \frac{1/\mathcal{L}}{-1/\mathcal{L}^2} = 0$, which is derived by using L'Hospital's Rule. Uniting this and the countinuity of $\Lambda(L_m)$, L_m can take $\Lambda(L_m) \in (0, \infty)$, therefore, we have

$$\frac{1}{\sigma} \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \frac{\beta - \mu}{\beta} \Rightarrow \lim_{X \to \infty} X^{\beta - \mu - \frac{\beta}{\sigma}} = \left\{ \begin{array}{c} \infty \\ 1 \\ 0 \end{array} \right. \Rightarrow L_m = \left\{ \begin{array}{l} 0 \\ L_m^* \in (0,1) \\ 1 \end{array} \right. . \quad (19)$$

It should be noted that the uniqueness of L_m^* (which is obtained by $L_m^* = \arg\{L_m | \Lambda(L_m) = \kappa_L\}$) is, at least, not shown in the Autor and Dorn (2013) setting, so multiple exixtences of L_m might happen under some specifications.

References

[1] Acemoglu, D., (2003) "Labor- and Capital-Augmenting Technical Change." *Journal of the European Economic Association*, Vol. 1, No. 1.

[2] Antras, P, (2004) "Is the U.S. Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity," B.E. Journal of Macroeconomics, Vol. 4, No. 1, pp. 1-36.

[3] Autor, D. H. and D. Dorn (2013) "The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market" *American Economic Review* 103(5): 1553-1597.

[4] Autor, D, Dorn, D, Katz, L, Patterson, C and Van Reenen J （2019）“ The Fall of the Labor Share and the Rise of Superstar Firms, ”Technical report, National Bureau of Economic Research, *(Quarterly Journal of Economics, Forthcoming)*, https://economics.mit.edu/files/12979

[5] Bell, D., (1974) *The Coming of Post-Industrial Society*. New York: Harper Colophon Books.

[6] Blanchard, Oliver （1997） "The Medium Run," Brookings Papers on Economic Activity, Vol. 2, pp. 89-158.

[7] Chirinko, Robert S. (2008) "Sigma: The Long and Short of It." *Journal of Macroeconomics* 30 (2): 671-86.

[8] Chirinko, Robert S., Steven M. Fazzari, and Andrew P. Meyer. (2011) “ A New Approach to Estimating Production Function Parameters: The Elusive Capital-Labor Substitution Elasticity. ” Journal of Business and Economic Statistics 29 (4): 587-94.

[9] Clark, C. (1941) *The Conditions of Economic Progress*, London, MacMillan & Co. Ltd.

[10] Cooley, T. F., and E. Prescott, (1995) "Economic Growth and Business Cysles" in T. F. Cooley ed. (1995) *Frontiers of Business Cycle Research*, Princeton University Press.

[11] Dao, Mai Chi, Mitali Das, Zsoka Koczan, and Weicheng Lian （2017）“ Why Is Labor Receiving a Smaller Share of Global Income? Theory and Empirical Evidence, ”Technical report, International Monetary Fund.

[12] Galbraith, J. K., (1958) *The Affluent Society*. Boston, Houghton Mifflin.

[13] Herrendorf, Berthold, Christopher Herrington, and Akos Valentinyi. (2013) “ Sectoral Technology and Structural Transformation. ” Center for Economic Policy Research Discussion Paper 9386.

[14] Hobraba, N., and S. Kuwahara. (2020) "An Economic Development Model with Education and Industriousness." mimeo

[15] Jones, C. I. (2016) "The Facts of Economic Growth," in J. B. Taylor, H. Uhlig ed., *Handbook of Macroeconomics*, Volume 2A, October 2016.

[16] Karabarbounis, L. and B. Neiman （2014）“ The Global Decline of the Labor Share, ”The Quarterly Journal of Economics, Vol. 129, No. 1, pp. 61?103, feb, DOI：http://dx.doi.org/10.1093/qje/qjt032.

[17] Keynes, J. M., (1930) " Economic Possibilities for Our Grandchildren, " in *Essays in Persuasion* (New York: Harcourt Brace, 1932), 358-373

https://assets.aspeninstitute.org/content/uploads/files/content/upload/Intro_and_Section_I.pdf

[18] Keynes, J. M., (1939)" Relative Movements of Real Wages and Output, " Economic Journal, Vol. 49, No. 193, pp. 34-51.

[19] Lawrence, Robert Z. （2015） " Recent Declines in Labor 's Share in US Income：A Preliminary Neoclassical Account, " National Bureau of Economic Research Working Paper 21296, DOI：http://dx.doi.org/10.3386/w21296.

[20] Maddison, A., (1982) *Phases of Capitalist Development.* Oxford: Oxford University Press.

[21] Oberfield, Ezra, and Devesh Raval. (2014) " Micro Data and Macro Technology. " National Bureau of Economic Research Working Paper 20452.

[22] Petty, W. (1690) *Political Arithmetick*, London: R. Cravel

[23] Schwellnus, C., M. Pak, P. Pionnier and E. Crivellaro （2018）" Labour share developments over the past two decades: The role of technological progress, globalisation and " winner-takes-most " dynamics "OECD Economics Department Working Papers, No. 1503

[24] Toffler, Alvin, (1980) *The Third Wave.* Bantam Books.

[25] Uzawa, H. (1961) "Neutral Inventions and the Stability of Growth Equilibrium." *Review of Economic Studies* 28: 117-24.

[26] Vries, de J. (1994) "The Industrial Revolution and the Industrious Revolution," *The Journal of Economic History* Vol. 54, No. 2, Papers Presented at the Fifty-Third Annual Meeting of the Economic History Association (Jun., 1994), 249-270.

[27] World Bank (1993) *The East Asian miracle: Economic growth and public policy.* New York: Oxford University Press.