The Rise of Information Societies and the Declining Labor Income Share

Shiro Kuwahara*

February 11, 2021

Abstract

Currently, the decline in the labor income share, which had been recognized in the past as stable in the long run, has been drawing a lot of attention. Furthermore, labor seems to have been polarizing into skilled and unskilled, or raw and intellectual, labor. Thus, we develop a simple model with endogenous raw and educated labor supplies and endogenous technological change. The model does not include transition dynamics, which stem from the constant resource allocation of goods, labor, and time. Long-run steady positive growth is realized with sufficiently high R&D profitability, a sufficiently highly educated labor supply, and sufficiently patient households. In an economy with a sufficiently lower contribution of raw labor, an increase in R&D activity increases the labor share, but it coinstantaneously expands the income difference between raw and educated labor.

Keywords: Raw labor; educated labor; R&D-based growth; poverty traps; long-run growth

JEL Classification: E00, O00, O41.

* *Affiliation:* University of Hyogo, *E-mail:* kuwahara@econ.u-hyogo.ac.jp

1 Introduction

Although the labor and capital income share has been considered stable during the economic growth process post World War II, many recent empirical studies, such as Blanchard (1997), OECD (2012), Karabarbounis and Neiman (2014), Dao et al.(2017), IMF (2017), and Autor et al. (2019), imply a declining labor income share. These studies report that the phenomenon of the fall in the labor share of income is not limited to advanced economies but is also evident in developing countries. Along with this phenomenon, we are experiencing colossal technological progress led by information technology. Thus, the declining labor income share seems to be caused by the emergent information society accompanied by the breaking down of the large middle class that comprises the industrial workers who are the core members of an "affluent society" (Galbraith 1958). Furthermore, because this contraction of the middle class has already yielded some poverty and various serious social problems in the affluence created by the information society, it is a vital agenda for economic research.

Although elasticities over 1 are necessary for connecting the observed decreasing price of capital goods with the declining labor income share, many studies estimate elasticities below 1.[1] To connect these empirically reported phenomena, the declining labor income share, and elasticities over 1, many studies (for example, Arpaia et al., 2009; Acemoglu and Autor, 2011; Elsby et al., 2013; and Grossman et al., 2017) differentiate two types of labor and focus on the difference in the elasticity of substitution between capital and labor and on capital against high- and low-skilled labor.

Many models have been developed with these two labor types. Lucas (1988) model with human capital accumulation has been extended to contain raw or unskilled labor by Ferrara and Guerrini (2010), among others.

Some studies focus on the functional differences between labor and human capital. Tran-Nam, Truong, and Tu (1995) introduce non-constant elasticity of substitution between labor and human capital, and Kuwahara (2006) addresses the discrimination between the economic roles of skilled and unskilled labor in economic development. Although the endowments of labor and human capital are exogenously given in these models, similar to Romer (1990), some models introduce the selection between raw or skilled labor at birth, as in Hori (2011).

Nowadays, obsolescence is acknowledged as a property of human capital. For example, De Grip and Van Loo (2002) point out the obsolescence

[1] For example, Antras (2004), Chirinko, Fazzari, and Meyer (2011), Herrendorf, Herrington, and Valentinyi (2013), Oberfield and Raval (2014), Chirinko and Mallick (2014), and Lawrence (2015).

caused by technological progress. Furthermore, Neuman and Weiss (1995) and Ramirez (2002) point out that people with higher education face the probability that innovations may render their education obsolete.

Thus, our model has a remarkable feature in that skilled labor is endogenously and instantaneously supplied; in other words, it is not accumulated. We insert this assumption into the Romer-type endogenous growth model and then allocate labor among three uses: raw labor employed in the final goods sector and educated labor employed in both final goods and the R&D sector.

Romer's (1990) seminal study incorporates R&D activities and long-run growth by assuming an exogenous fixed labor (unskilled labor) and human capital (skilled labor) supply and concludes that a small endowment of R&D input (labor or human capital), which is assumed to be fixed, yields a steady state without positive long-run growth. By contrast, this study incorporates the endogenous supply of two types of labor into the Romer model of endogenous technological change: raw and educated labor. The model used in our study relates the efficiency and supply of education and these two types of labor to the long-run growth rate and examines the cause of poverty traps.

We also omit capital and durable goods, which would introduce transitional dynamics. Thus, we simplify the model by eliminating the two factors that generate transitional dynamics, thereby enabling us to focus on the absence or presence of growth and the decline in the labor income share (caused by the education labor allocation).

This study finds, first, that both types of labor are always supplied, but R&D activities that utilize skilled labor are not always executed; thus, our model contains two types of steady states that comprise R&D-based growth and poverty traps. Long-run steady positive growth is realized when there is sufficiently high R&D profitability, a sufficiently highly educated labor supply, high educational efficiency, and sufficiently patient households, (i.e., households with sufficiently high subjective discount rates), for example, in the case with high innovation efficiency and high monopoly power. The opposite also holds.

Second, we derive a lemma in which an increase in R&D activities stimulates the growth rate, which results in the wage premium of educated labor, and the increase in R&D activities also yields an increase in the educated labor share. Furthermore, if an economy has a sufficiently lower contribution of raw labor, the increment of R&D activities increases the labor share, but in this case, the income difference between raw and educated labor is coinstantaneously generated.

The remainder of this paper is organized as follows. The next section describes the basic model. Section 3 derives steady-state equilibria. Section

4 presents the steady state of the command economy and the optimal policy implications of the model are obtained. Section 5 concludes the paper.

2 The Model

The model used in this study follows that proposed in Romer (1990). It includes three sectors and four production factors. The three sectors include a final goods sector, an intermediate goods sector, which comprises a continuum of intermediate goods firms indexed by $i \in [0, A]$, and an R&D sector that creates new varieties of goods by employing existing knowledge and educated labor. The four production factors are physical capital (denoted as "capital") K; knowledge A, which is measured in terms of the variety of intermediate goods; and two types of human resources, one of which is unskilled or raw labor ("raw labor") L endowed at one unit per household, and skilled or intellectual labor ("educated labor")) H, which is supplied through education. Labor, skilled labor, and capital are direct inputs used in the final goods sector. Knowledge functions as an indirect input in the production of final goods. Intermediate goods firms monopolistically supply intermediate goods using capital. Monopoly power is conferred by a patent, which an intermediate goods firm obtains from an R&D firm. The R&D firms produce new varieties of intermediate goods using educated labor and existing knowledge as inputs. The cost of R&D activities is covered by the return on the sale of the patent.

There are two additional factors: one is the constantly decreasing cost of intermediate goods production, which reflects the constantly improving informational technology, for example, the constantly decreasing price of personal computers (PCs) in Autor and Dorn (2013), which we introduce as the persisting decreasing cost of intermediate goods, and the other is the education sector where the input is unskilled labor (also low labor or labor) and which yields skilled labor (also educated labor or human capital).

2.1 Goods Production

Final goods are assumed to be competitive and are produced using labor and intermediate goods. The specified production function is defined as follows:

$$Y = \left(L^{\beta} H_Y^{(1-\beta)}\right)^{1-\alpha} \int_0^A x(i)^{\alpha} di, \quad 0 < \alpha, \beta < 1, \tag{1}$$

where Y, L, H_Y, A, and $x(i)$, respectively indicate the amount of final goods, the labor used in producing final goods, the educated labor used in final goods, the variety index, and the intermediate goods used in sector-i. In addition, α and $1-\alpha$ denote the shares of capital and human resources, respectively. Capital is used to produce intermediate goods, and human resources are constituted by raw and educated labor. The shares of raw and educated labor in final goods production are represented as β and $1-\beta$, respectively. From Eq. (1), the first-order conditions indicate:

$$w_L = \frac{\beta(1-\alpha)Y}{L}, \quad w_Y = \frac{(1-\beta)(1-\alpha)Y}{H_Y}, \quad \text{and} \quad p(i) = \alpha(L^\beta H_Y^{1-\beta})^{1-\alpha} x(i)^{\alpha-1}, \tag{2}$$

where w_L, w_Y, and $p(i)$ denote the raw labor wage, educated labor wage offered by the final goods sector, and the price of the ith intermediate goods.

An intermediate goods firm i is a firm that possesses a permanent patent on the intermediate goods used in sector i. Consequently, the intermediate goods firm can supply the ith intermediate good monopolistically.

Following Romer (1990), we assume that one unit of intermediate goods requires η units of final goods, and we additionally assume that this η is decreasing at an exogenously given rate $-\gamma$, which captures the information technology in this research. Autor and Dorn (2013) assume that these are PCs and durable goods (capital), and we also assume they are PCs but rather non-durable goods (input). Then, we can suppose that the emergent information society is an event of the increment of γ from 0 (or small) to positive.

Because the final good is adopted as a numeraire, its price is 1. Therefore, the profit of the ith intermediate goods firm ($\pi^m(i)$) is given as $\pi^m(i) = p(i)x(i) - \eta x(i)$. Using the inverse demand function for an intermediate good $x(i)$ given in Eq. (2), we obtain the demand and price of intermediate goods in a symmetric equilibrium as

$$x = x(i) = \left(\frac{\alpha^2}{\eta}\right)^{\frac{1}{1-\alpha}} L^\beta H_Y^{1-\beta}, \quad \text{and} \quad p = p(i) = \frac{\eta}{\alpha}, \tag{3}$$

where x and p denote a common production cost and price for symmetric intermediate goods, respectively. The intermediate goods are assumed to be produced from η units of final goods. Therefore, the aggregate final good input to the intermediate sector X is defined as

$$X := \int_0^A \eta\, x(i) di = \eta A\, x. \tag{4}$$

By using the definition of Eq. (3), eliminating x from Eq. (1) yields the aggregate product as follows:

$$Y = \left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}} AL^{\beta}H_Y^{1-\beta}. \tag{5}$$

From Eqs. (3), (4), and the resource constraints of final goods $Y = C + X$, X, and C are given as

$$X = \alpha^2 Y, \quad \text{and} \quad C = (1-\alpha^2)Y. \tag{6}$$

Using the variable Y, the profit of a firm in the intermediate goods sector can be rewritten as

$$\tilde{\Pi}^M = \alpha(1-\alpha)\frac{Y}{A}, \tag{7}$$

where $\tilde{Z} := Z/A$ denotes the efficiency-adjusted value for an aggregate value Z.

Equations derived in (5) and (6) immediately imply that

$$\frac{\dot{Y}}{Y} = \frac{\dot{X}}{X} = \frac{\dot{C}}{C} = \frac{\alpha}{1-\alpha}\gamma + \frac{\dot{A}}{A} + \beta\frac{\dot{L}}{L} + (1-\beta)\frac{\dot{H}_Y}{H_Y}, \quad \text{for} \quad \forall t. \tag{8}$$

2.2 The R&D Sector

In this model, innovation is assumed to be the discovery of a new intermediate good design that is subsequently added to the existing set of intermediate goods (A). Therefore, the accumulation of newly designed intermediate goods can be measured by the increment of knowledge $\dot{A}$.

These R&D activities accumulate new knowledge that is measured by the design of intermediate goods. The aggregate accumulation of new knowledge is denoted as $\dot{A}$.

Free entry into R&D activities is assumed. Consequently, if $\tilde{\Pi}^R > 0$, where $\tilde{\Pi}^R$ denotes the profit of an R&D firm, then an infinite amount of educated labor would be input in R&D activities. This cannot hold in equilibrium. By contrast, if $\tilde{\Pi}^R < 0$ holds, then investment in R&D is less profitable. Consequently, the R&D input stops and an equilibrium without R&D ($H_A = 0$) occurs. However, if $\tilde{\pi}^R = 0$, then a positive amount of educated labor is devoted to R&D and the market would be in equilibrium. These are summarized as follows:

$$\tilde{V}\dot{A} \leq w_A H_A, \quad \text{for} \quad H_A \geq 0, \tag{9}$$

where $\tilde{V}$ and w_A respectively show the value of a firm with a patent, that is, the value of one successful R&D, and the wage offered in the R&D sector.

The value of R&D $\tilde{V}$ is equal to the price of the design. The present value of this stream represents the R&D value:

$$\tilde{V} := \int_0^\infty \tilde{\Pi}^M(\tau) e^{-\int_0^\tau r(s)ds} d\tau.$$

Time derivation of the above yields

$$r\tilde{V} = \dot{\tilde{V}} + \tilde{\Pi}^M. \tag{10}$$

It should be noted that because each variety of intermediate goods is patented by a firm, and the number of patents becomes the efficiency of the economy, the value of one firm equals the efficiency-adjusted value of all firms.

We assume that, in the innovation process, R&D firms enjoy the free use of knowledge, which is measured using the entire stock of intermediate goods variety (A) and the educated labor (H_A) by paying wage w_A. The former appears plausible only because knowledge is a non-rival commodity, which is emphasized in Romer's (1990) endogenous growth model. The Romer-type R&D function is given as follows:

$$\dot{A} = \delta \frac{AH_A}{\Omega}.^2 \tag{11}$$

where Ω denotes the R&D difficulty index introduced by, for example, Segerstrom (1998). Substituting Eqs. (10) and (11) into $\tilde{V}\dot{A} = w_A H_A$, we obtain the following condition of positive R&D activities:

$$\delta \frac{\tilde{V}A}{\Omega} = w_A, \tag{12}$$

where $V := A\tilde{V}$ is the aggregate value of R&D, and $\tilde{v} := \frac{V}{AN} = \frac{\tilde{V}}{N}$ denotes the per capita holding of a firm's stock. If the equation $\tilde{v} = w_A/\delta A$ does not hold, R&D activities are assumed not to be executed.

[2]This type of R&D technology is different from the Jones technology as advocated by Jones (1995). For example, it is specified as $\dot{A} = \delta A^a L_A^b$. This technology fails to analyze the no-growth traps. The marginal productivity of R&D input diverges infinitely when the R&D input tends to 0; $\lim_{L_A \to 0} \partial \dot{A}/\partial L_A = \infty$. The marginal productivity of R&D input increases to infinity when the R&D input decreases to 0, and $\lim_{L_A \to 0} \partial \dot{A}/\partial L_A = \infty$. Therefore, an economy with Jones technology always provides a steady state with positive R&D activities when the R&D input has a positive growth rate, for example, a positive population growth in the case where R&D is executed by labor input. This study specifically addresses both steady states with and without R&D under positive population growth. Therefore, throughout the study, we adopt the specification of (11), which can relate the share of educated labor devoted to R&D and the education level of labor with the GDP growth rate of the economy. Furthermore, Kuwahara (2019) discusses the existence of poverty traps and the assumption of R&D efficiency.

In equilibrium, the arbitrage condition with wage rates for educated labor equates w_Y and w_H through allocation of educated labor, which determines the share of education labor allocation u^*. If $w_Y > w_A$ for $\forall u$, then $s = 1$, and if $w_Y < w_A$ for $\forall u$, then $s = 0$. However, if all educated labor is devoted to the R&D sector, the wage rate w_A diverges to ∞ because w_Y in (2). Therefore, $u = 0$ cannot be an equilibrium. These are summarized as

$$\left.\begin{array}{l} u\text{: inner solution} \\ u = 1 \end{array}\right\} \Longleftrightarrow w_A \left\{ \begin{array}{c} = \\ < \end{array} \right\} w_Y. \tag{13}$$

Therefore, the human capital wage w_H is determined by $w_H = w_Y$ because the final goods are always produced. Furthermore, if positive R&D activities are undertaken, then $w_H = w_Y = w_A$ holds. Section 3.2 presents a derivation of the determination of R&D activities from this arbitrage condition for wages.

2.3 Households

2.3.1 Optimization for Consumption and Saving Allocation

The representative household has one unit of labor force. $l(\in [0,1])$ unit of labor force is supplied as raw labor, and the residual part $1 - l$ is supplied to education. Education produces educated labor, which is denoted as h. The household obtains utility from their consumption of final goods (denoted as c) and has a constant intertemporal elasticity of substitution, $1/\sigma$. From these assumptions, the maximization problem of the household is specified as

$$\begin{aligned} \max \quad U_t &= \int_t^\infty \frac{c(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho(s-t)} ds, \quad \rho > 0, \quad \sigma > 0, && (14) \\ \text{s.t.} \quad \dot{a}(t) &= r(t)a(t) + w_L(t)l_t(t) + w_H(t)h(t) + c(t) - na(t), && (15) \\ \text{and} \quad h_t &= \Psi(l(t)), \quad \Psi'(\cdot) < 0, && (16) \end{aligned}$$

where ρ, a, and n respectively denote the subjective discount rate, per capita stock holding, and exogenously assumed constant population growth rate $(n := \dot{N}/N)$, where N denotes the population scale. It should be noted that $L = lN$ and $H = hN$ hold under the assumption of a representative household assumption.

From the conditions above, a Euler equation is obtained as

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}(r - n - \rho). \tag{17}$$

The transversality condition (TVC) is given as

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)a(t) = 0, \tag{18}$$

where $\lambda(:= c^{-\sigma})$ is a shadow price of capital stock k.

The maximization problem (14)-(16) yields the optimal condition

$$\Psi'(l) = \frac{w_L}{w_H} = \frac{(1-\alpha)\beta Y/L}{(1-\alpha)(1-\beta)Y/H_Y} = \frac{\beta}{1-\beta}\frac{l}{uh}, \tag{19}$$

where the third equation of (19) is derived from (2), and u in the fourth equation is defined as: where $u := \frac{H_Y}{H}$, namely, u denotes the final goods allocation share of human capital (skilled labor).

This condition equates the earning rate of both skilled and unskilled (or educated and raw) labor.

2.3.2 Optimization for Human Resource Allocation

Because we assume that H is used either in final goods production or in the R&D sector, using the ratio of educated labor devoted to the management process against the aggregate educated labor, $u = H_Y/H$, we can immediately obtain $1-u = \frac{H_A}{H}$. Furthermore, we assume the per capita value of H as $h = \frac{H}{N}$.

The representative household determines the supplies of raw and educated labor forces by considering their wage rates. This optimizing condition is given in (19). Throughout the paper, we specify the education function as

$$h = \Psi(l) := b(1-l)^{\phi}, \quad b > 0, \quad \phi \in (0,1) \tag{20}$$

where θ and ϕ represent the level and marginal educational efficiency, respectively. It is noteworthy that because of l, $\phi \in (0,1)$, for a higher ϕ, the educated human capital or product of education, the given input $1-l$ is lower; therefore, a smaller ϕ implies greater efficiency.

Using Eqs. (2), (19), and (20), the optimal educated labor investment is given as

$$\frac{\beta u}{(1-\beta)l} = \phi(1-l)^{-1}. \tag{21}$$

Solving this equation with respect to l yields raw and educated labor supplies as

$$l(u) = \frac{\beta u}{\beta u + \phi(1-\beta)}, \quad \text{and} \quad h(u) = b\left(\frac{\phi(1-\beta)}{\beta u + \phi(1-\beta)}\right)^{\phi}. \tag{22}$$

Thus, we have the following lemma.

Lemma 1 *(22) shows that the representative household supplies constant labor and educated labor against the given human capital allocation rate u.*

Using (22), the aggregate values are given as

$$L = l(u)N, \quad H_Y = uh(u)N, \quad \text{and} \quad H_A = (1-u)h(u)N. \tag{23}$$

Using (19) and (22) and the notation given in (23), we obtain the wage rate for educated and raw labor $\psi(u) := w_H/w_L$ as follows:

$$\psi(u) = \frac{1-\beta}{b\phi^{\phi}(1-\beta)^{\phi}}[\beta u + \phi(1-\beta)]^{\phi-1}. \tag{24}$$

Thus, we obtain:

$$\psi'(u) < 0, \tag{25}$$

and, therefore, a decrease in u increases the wage premium of educated labor.

From (22), we can confirm the skilled and unskilled labor supply and growth rate. The obtained effects of the change in skilled labor allocation on the macroeconomic valuables $h(u)$ and $l(u)$ are as follows:

$$l'(u) = \frac{\beta\phi(1-\beta)}{\left\{\beta u + \phi(1-\beta)\right\}^2} > 0, \tag{26}$$

$$h'(u) = -\frac{\beta\phi h(u)}{\beta u + \phi(1-\beta)} < 0. \tag{27}$$

2.4 Macroeconomic variables

Here, we relate the growth rate and human capital allocation. From (11) and $H_A = (1-u)H$, we obtain

$$\frac{\dot{A}}{A} = \delta\frac{(1-u)H}{\Omega} \tag{28}$$

From (23) and the equation above, it is necessary for Ω, at least in a steady state, to grow at the same rate as H and, therefore, N. For simplicity, we assume that $\Omega = N$.

Combining Eqs. (11), (22), (23), and the assumption $\Omega = N$, we obtain the growth rate of the number of intermediate goods, which is the TFP (total factor productivity) growth rate, as the following function of u:

$$g_A(u) = \delta(1-u)h(u), \tag{29}$$

where the growth rate of variable Z is written as $g_Z(:= \dot{Z}/Z)$ in this study.

From these results and Eqs. (23) and (29), we can confirm the effects of changes in u on macroeconomic valuables H_Y, H_A, and g_A, as follows:

$$H'_Y(u) = h(u)N\frac{\beta u(1-\phi)+\phi(1-\beta)}{\beta u+\phi(1-\beta)} > 0, \tag{30}$$

$$H'_A(u) = -h(u)N + (1-u)h'(u)N < 0, \tag{31}$$

$$g'_A(u) = -\delta h(u) + \delta(1-u)h'(u) < 0. \tag{32}$$

An increase in u increases $h(u)$ and H_Y, but always decreases H_A, which decreases the growth rate.

From (25), (27), and (29), we obtain the following lemma:

Lemma 2 *An increase in R&D activities stimulates the growth rate and causes the wage premium of educated labor.*

By substituting Eqs. (22) and (23) into (2), (7) and (5), we obtain the efficiency-adjusted per capita valuables as follows:

$$\tilde{y} = \left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}} l(u)^\beta \big(uh(u)\big)^{1-\beta}, \quad \tilde{\pi}^M = \alpha(1-\alpha)\left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}} l(u)^\beta \big(uh(u)\big)^{1-\beta},$$

$$\text{and} \quad w_H = (1-\beta)(1-\alpha)\left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}} Al(u)^\beta \big(uh(u)\big)^{-\beta}. \tag{33}$$

where $\tilde{z}$ denotes the knowledge-adjusted per capita variable for variable Z, namely, $\tilde{z} := \frac{Z}{NA}$. It is useful to express the system in terms of variables that will be constant in a steady state. The variables denoted as $\tilde{z}$ are constant in the steady state.

3 Dynamics and Steady States

In this section, we derive dynamic equations and steady states, which are defined as states in which all variables are growing at constant (not necessarily the same and including zero) rates.

3.1 Dynamic Equations

From (12), (13), and (33), we obtain

$$\tilde{V}(t) = \frac{(1-\beta)(1-\alpha)}{\delta}\left(\frac{\alpha^2}{\eta(t)}\right)^{\frac{\alpha}{1-\alpha}} N(t)l\big(u(t)\big)^\beta \left(u(t)h\big(u(t)\big)\right)^{-\beta}. \tag{34}$$

Differentiating $\tilde{V}$ with respect to time, we have the dynamic equation of the firm's value, $r(t)\tilde{V}(t) = \dot{\tilde{V}}(t) + \tilde{\Pi}(t)^M$. Substituting Eqs. (33), (34), $\dot{\eta}/\eta = -\gamma$, and $\dot{N}/N = n$ into this dynamic equation for the firm's value yields the following equation:

$$\frac{\dot{\tilde{V}}}{\tilde{V}} = n + \frac{\alpha}{1-\alpha}\gamma + \Phi_1 \frac{\dot{u}}{u} = r - \frac{\alpha\delta}{1-\beta} u h(u), \tag{35}$$

where $\Phi_1 := \beta \left[\frac{l'(u)u}{l(u)} - 1 - \frac{h'(u)u}{h(u)} \right] = -\frac{\beta u(1+\phi)}{\beta u + \phi(1-\beta)} < 0$. Here, we can immediately give the following notations:

$$\varepsilon_{ul} := \frac{l'(u)u}{l(u)} = \frac{\phi(1-\beta)}{\beta u + \phi(1-\beta)} \in (0,1), \text{ and } \varepsilon_{uh} := -\frac{h'(u)u}{h(u)} = \frac{\beta\,\phi\,u}{\beta u + \phi(1-\beta)} \in (0,1).$$

From Eqs. (8), (17), (22), and (29), the Euler equation is expressed as

$$\Phi_2 \frac{\dot{u}}{u} = r - n - \rho - \sigma \left(\frac{\alpha}{1-\alpha}\gamma + \delta(1-u)h(u) \right). \tag{36}$$

where $\Phi_2 := \sigma \left[\beta\varepsilon_{ul} + (1-\beta)(1-\varepsilon_{uh}) \right] = \frac{\sigma(1-\beta)}{\beta u + \phi(1-\beta)} \left\{ \phi + (1-\phi)\beta u \right\} > 0$. The third term is derived by using Eqs. (23) and (35).

By eliminating $r - n$ from Eqs. (35) and (36), we have the dynamics of u as a function of u as follows:

$$(-\Phi_1 + \Phi_2)\frac{\dot{u}}{u} = \Gamma(u) - \left[\rho - (1-\sigma)\frac{\alpha}{1-\alpha}\gamma \right], \tag{37}$$

where $\Gamma(u^*) := \left[\left(\frac{\alpha}{1-\beta} + \sigma \right) u^* - \sigma \right] \delta h(u^*)$. Because all the dynamics that we have obtained are combined into this equation, the dynamics of the economy are depicted by this dynamic equation on u.

$\Phi_1 < 0$ and $\Phi_2 > 0$ immediately imply $-\Phi_1 + \Phi_2 > 0$.

3.2 Steady State

To obtain constant growth rates in the steady state, Eq. (29) implies that a constant u (denoted as u^*) is necessary, which implies that $\dot{u} = 0$, and the constant u^* pins down $l(u)$ and $h(u)$ at $l^* = l(u^*)$ and $h^* = h(u^*)$. Thus, uniting Eqs. (5), (6), (8), and u^* yields the following growth rate condition in a steady state:

$$g^* := g_Y = g_C = g_A + n + \frac{\alpha}{1-\alpha}\gamma, \quad \text{and} \quad g_y = g_c = \delta(1-u^*)h(u^*) + \frac{\alpha}{1-\alpha}\gamma. \tag{38}$$

Uniting the condition of steady state $\dot{u} = 0$, Eq. (35) implies the equilibrium interest rate as follows:

$$r^* = n + \frac{\alpha}{1-\alpha}\gamma + \frac{\alpha\delta}{1-\beta}u^* h(u^*). \tag{39}$$

From Eqs. (18) and (38), the TVC is calculated as

$$\rho > (1-\sigma)\left(g_A(u^*) + \frac{\alpha}{1-\alpha}\gamma\right).$$

Substituting $\dot{u} = 0$ into (37) yields the following expression:

$$\Gamma(u^*)\left\{\begin{array}{c} = \\ < \end{array}\right\}\rho - (1-\sigma)\frac{\alpha}{1-\alpha}\gamma \Longleftrightarrow \left\{\begin{array}{l} u^* \in (0,1) \\ u^* = 1 \end{array}\right. \tag{40}$$

This equation is a key to determining the properties of the steady state, because this condition determines the equilibrium allocation of skilled labor between final goods and R&D activities, which determines the steady-state values of the GDP growth rate through (29), the efficiency-adjusted per capita GDP level through (33), and the growth rate through (37), and so on.

To derive this equation's properties, we differentiate it and obtain:

$$\Gamma'(u) = \frac{\delta h(u)}{\beta u + \phi(1-\beta)}\left[\left(\frac{\alpha}{1-\beta} + \sigma\right)\left\{\beta u(1-\phi) + \phi(1-\beta)\right\} + \beta\phi\right] > 0, \tag{41}$$

and substituting $u = 0$ into $\Gamma(u)$ yields $\Gamma(0) < 0$, we obtain Fig.1, where $\underline{u} := \arg\{u|\Gamma(u) = 0\}$ is easily checked as $\underline{u} = \frac{\sigma}{\frac{\alpha}{1-\beta}+\sigma} \in (0,1)$.

Uniting the properties of $\Gamma(u)$ and u^* into Eq. (37), we obtain the dynamics of u as unstable, and the variable u is jumpable. A unique solution under the rational expectations is to maintain u^* from the initial period to the infinite future.

Result 1 *The equilibrium value of u^* is uniquely determined by deep parameters $\{\alpha, \beta, \delta, \sigma, \rho, \gamma\}$. Unambiguous results are obtained as $\frac{\partial u^*}{\partial \gamma} < 0$, $\frac{\partial u^*}{\partial \rho} > 0$, and $\frac{\partial u^*}{\partial \delta} < 0$. The economic path with rational expectations is to maintain u^* for all times.*

Our main concern of the determinant of u is the constantly decreasing PC price. The effect of γ depends on the condition $\sigma\left\{\begin{array}{c} > \\ < \end{array}\right\}1$. Following Vissing-Jorgenson & Attanasio (2003) and Xu (2017), we assume $IES > 1$, namely $\sigma < 1$. Then, we obtain the following result as follows:

Corollary *Under the assumption of $\sigma < 1$, we have the negative affect of the increase of γ on u^*. Therefore, through $g_A(u)$, an increase in the decreasing rate of the PC price affects the equilibrium growth rate positively.*

3.3 Emergence of Poverty Traps

From the previous section, the intersection of $\Gamma(u)$ and ρ gives the steady state. For $\rho > 0$, we obtain:

$$\Gamma(u) \left\{ \begin{array}{c} > \\ < \end{array} \right\} 0 \Longleftrightarrow u \left\{ \begin{array}{c} > \\ < \end{array} \right\} \underline{u},$$

where, because $\underline{u} \in (0,1)$ and $\rho > 0$, the intersection of $\Gamma(u)$ and ρ, u^* always exists in the interval $u^* > \underline{u}$. Therefore, when $\Gamma(1) < \rho$ holds, then $\bar{u} = 1$ can be an equilibrium. In this case, a no-R&D equilibrium emerges. We call the steady state in a no-R&D equilibrium a "poverty trap."

Summarizing these conditions of the steady state with labor market equilibrium conditions yields the following condition:

$$\begin{aligned} h(1) = b\left(\frac{\phi(1-\beta)}{\beta+(1-\beta)\phi}\right)^{\phi} \left\{ \begin{array}{c} > \\ < \end{array} \right\} \frac{1-\beta}{\alpha\,\delta}\left(\rho-(1-\sigma)\frac{\alpha}{1-\alpha}\gamma\right) \\ \Longleftrightarrow \delta \left\{ \begin{array}{c} > \\ < \end{array} \right\} \frac{(1-\beta)\rho}{\alpha b}\left(\frac{\beta+(1-\beta)\phi}{\phi(1-\beta)}\right)^{\phi} \\ \Longleftrightarrow \left\{ \begin{array}{c} \text{Steady Growth Path} \\ \text{Poverty Traps} \end{array} \right. . \end{aligned} \tag{42}$$

Because $h(1)$ is the educated labor supply for the no R&D equilibrium case, T is a threshold value of emerging poverty traps related to the supply of educated labor. This condition also implies that poverty traps emerge from low b, α, and δ and high ρ. Because b, δ, and ρ denote education efficiency, R&D efficiency, and temporal endurance, respectively, the results are intuitive.

From Eq. (42), lower θ, α, and δ, and higher ρ generate poverty traps. The higher efficiency of education level, capital production, and R&D, and a lower subjective discount rate derive long-run steady growth.

(42) is transformed into the following expression:

$$P(\phi) := \frac{\phi^{\phi}(1-\beta)^{\phi-1}}{(\beta+(1-\beta)\phi)^{\phi}} \left\{ \begin{array}{c} > \\ < \end{array} \right\} \frac{\rho}{\alpha\delta b}$$

Differentiating $P(\phi)$ yields

$$\frac{dP(\phi)}{d\phi} = P(\phi)\underbrace{[1-\xi+\log\xi]}_{f(\xi)},$$

where $\xi := \frac{(1-\beta)\phi}{\beta+(1-\beta)\phi} \in (0,1)$. Because $f(\xi)$ has the following properties of

$$f'(\xi) = -1 + \frac{1}{\xi} > 0, \quad \text{and} \quad \lim_{\xi \to 1} f(\xi) = 0,$$

$f(\xi)$ is negative for $\xi \in (0,1)$, which provides $P'(\phi) < 0$. This result implies that a large ϕ can derive $P(\phi) < \rho/(\theta\alpha\delta)$. A smaller ϕ increases the supply of educated labor. Therefore, a smaller ϕ is necessary to escape poverty traps.

Eq. (42) is also transformed into

$$B(\beta) := \frac{(1-\beta)^{\phi-1}}{(\beta + (1-\beta)\phi)^{\phi}} \left\{ \begin{array}{c} > \\ < \end{array} \right\} \frac{\rho}{\alpha\delta b \phi^{\phi}}.$$

Differentiating $B(\beta)$ yields

$$\frac{dB(\beta)}{d\beta} = \frac{\beta(1-\phi)^2(1-\beta)^{\phi-1}}{(\beta+(1-\beta)\phi)^{\phi}+1} > 0,$$

Therefore, a large β can derive $B(\beta) > \rho/(b\phi^{\phi}\alpha\delta)$. A higher share of raw labor in final goods production is necessary for positive growth.

To summarize, we obtain the following result:

Result 2 *Large b, β, α and δ, and small ϕ and ρ generate long-run positive growth.*

Long-run steady positive growth is realized when sufficiently high R&D profitability (i.e., the case with high innovation efficiency δ and high monopoly power α), sufficiently highly educated labor supply (corresponding to a higher b and lower ϕ, which is the case of a larger supply of skilled labor), and sufficiently patient households capture a higher subjective discount rate ρ. Furthermore, long-run steady zero growth is realized under the inverse case.

4 Labor Share and Capital Share

In this section, we discuss labor and capital share. The factors that receive factor distribution are raw labor (L), educated labor (H), and capital, introduced as financial assets in this study (V). Thus, in this study GDP Υ is given as $\Upsilon := w_L L + w_H H + rV$, and it should be noted that Υ is different from the final goods production Y obtained in (5)[3]. Because w_L and w_H are

[3]This is because Y contains intermediate input. Furthermore, R&D adds value to the economy.

given in (2) and V and r are, respectively, obtained in Eqs. (34) and (39), we have the following equations:

$$\underbrace{\Upsilon}_{\text{GDP}} = \underbrace{\beta(1-\alpha)Y}_{\text{Raw Labor Distribution}(w_L L)} + \underbrace{\frac{(1-\beta)(1-\alpha)}{u}Y}_{\text{Educated Labor Distribution}(w_H H)}$$
$$+ \underbrace{\left[\alpha + \frac{1-\beta}{\delta\, u\, h(u)}\left(n + \frac{\alpha}{1-\alpha}\gamma\right)\right](1-\alpha)Y}_{\text{Capital Distribution}(rV)} \tag{43}$$

We respectively abbreviate raw labor distribution, educated labor distribution, and capital distribution as RLD, SLD, and CD. These terms all contain $(1-\alpha)Y$, therefore, to calculate the shares, we can treat the following variables:

$$G\hat{D}P(:= \hat{\Upsilon}) = \frac{\Upsilon}{(1-\alpha)Y} = \beta + \frac{1-\beta}{u} + \alpha + \frac{1-\beta}{\delta\, u\, h(u)}\left(n + \frac{\alpha}{1-\alpha}\gamma\right),$$
$$R\hat{L}D = \beta, \quad E\hat{L}D = \frac{1-\beta}{u}, \quad \hat{CD} = \alpha + \frac{1-\beta}{\delta\, u\, h(u)}\left(n + \frac{\alpha}{1-\alpha}\gamma\right)$$

It should be noted that $\hat{\Upsilon}$ is a function of u. Now, to check the effect of the change in u on $\hat{\Upsilon}$, we derive and obtain the following:

$$\frac{d\hat{\Upsilon}(u)}{du} = -\frac{1-\beta}{\delta u^2}\left[\delta + \frac{1-\varepsilon_{uh}}{h(u)}\left(n + \frac{\alpha}{1-\alpha}\gamma\right)\right] < 0. \tag{44}$$

This property stems from the positive increasing R&D input (it should be noted that an increased u is connected with low R&D through a low-skilled labor supply) effects on the relative GDP.

Thus, we immediately obtain the property of the raw labor share (RLS)$(:= R\hat{L}D/G\hat{D}P)$ as follows:

$$\frac{dRLS(u)}{du} = -\frac{\beta}{\hat{\Upsilon}(u)^2}\frac{d\hat{\Upsilon}(u)}{du} > 0 \tag{45}$$

Thus, we can depict an exogenous labor share decreasing through some causes under an increasing R&D.

After some calculation, we obtain the property of skilled (educated) labor share (ELS)$(:= S\hat{L}D/G\hat{D}P)$ as follows:

$$\frac{dELS(u)}{du} = -\frac{1-\beta}{\Upsilon(u)^2 u^2}\left[\alpha + \beta + \frac{(1-\beta)\varepsilon_{uh}}{\delta u h(u)}\left(n + \frac{\alpha}{1-\alpha}\gamma\right)\right] < 0. \tag{46}$$

Thus, the emergent information society characterized by the persisting decreasing price of PCs, which corresponds with the persisting decreasing cost of η in this study, is shown to decrease u, which increases SLS and decreases RLS. Thus, we obtain the following result:

Result 3 *From Eqs. (45) and (46), we obtain the obtain the wage polarization between raw and educated labor caused by the emergent information society.*

Next, we check the labor share ($LS = RLS + ELS$). From Eqs. (45) and (46), we obtain:

$$\frac{dLS(u)}{du} = \frac{1-\beta}{\delta\,\Upsilon(u)^2 u^2 h(u)}\left[-\alpha\,\delta\,h(u) + N(u;n,\gamma,\alpha,\beta,\phi)\right] \tag{47}$$

where $N(u;n,\gamma,\alpha,\beta,\phi) := \left(n + \frac{\alpha}{1-\alpha}\gamma\right)\frac{\beta^2 u\,(1-\phi)}{\beta u+\phi(1-\beta)}$, and we can easily obtain $N'(u) > 0$. As Figure 2 shows, if $N(1) > \alpha\delta h(1)$ holds, there exists $\underline{u}$ and for $u \in (\underline{u}, 1)$, $LS'(u) > 0$ holds. Thus, we cannot conclude from this study's results that the emergence of the information society is *always* the cause of the decrease in labor share. However, the following can be said: At least, for small δ and b, and large γ, $LS'(u) > 0$ holds, which implies that a decrease in u, driven by the emergent information society captured by the constantly decreasing cost $\gamma(> 0)$, generates a decrease in the total labor share through the declining u.

Thus, we cannot conclude from this study's results that the emergence of the information society is *always* the cause of the decrease in labor share. However, the following can be said: At least, for small δ and b, and large γ, $LS'(u) > 0$ holds, which implies that a decrease in u, driven by the emergent information society captured by the constantly decreasing cost $\gamma(> 0)$, generates a decrease in the total labor share through the declining u.

Result 4 *In the case of the lower efficiency effect of intermediate goods productivity on final goods production, R&D activity, and educational performance, emerging informational industrialization would reduce not only the raw labor share of income, but also the total labor share.*

5 Conclusions

This study incorporated endogenous raw and educated labor supplies into Romer's (1990) model with decreasing intermediate good cost, and provides some relationships between endogenous labor supply and some economic growth phenomena.

The results obtained are summarized as follows: Because we assume that human capital (skilled labor) is produced by the educational technology with the Inada conditions, both types of labor are always supplied, but R&D activities that utilize skilled labor are not always executed; thus, our model contains two types of steady states that consist of R&D-based growth and poverty traps. If R&D activities are profitable, an economy grows through the promotion of endogenous technological progress. If not, the economy allocates no input for R&D; consequently, no technological progress occurs. It remains an economy in a no-growth situation. This profitability is related to the adequate supply of educated labor, rather than the efficiency of education.

We derive the conditions that generate long-run endogenous growth. If education efficiency, human capital share, and intermediate goods share are sufficiently high, and there is sufficiently educated labor and sufficiently patient households, then the economy grows steadily. The inverse conditions produce a stationary state with zero growth.

Next, we derive a lemma in which an increase in R&D activities stimulates the growth rate, resulting in the educated labor wage premium. Further, the increase in R&D activities also yields an increase in the educated labor share of income, and if an economy has a sufficiently lower contribution of raw labor, the increment of R&D activities increases the labor share of income; however, currently, the income difference between raw and educated labor is coinstantaneously generated.

These results are still preliminary; for example, the labor share of income is constant in this study, therefore the decrease in labor share is exogenous. It is necessary, therefore, to progress our research to analyze the endogenously caused decrease in labor share.

Acknowledgement This work was conducted with financial support from JSPS KAKENHI Grant Number 15K03360, JSPS KAKENHI Grant Number 26380348, and the grant from Kobe Academic Park Association for the Promotion of Inter-University Research and Exchange.

References

[1] Acemoglu, D. and Autor, D. (2011) " Skills, Tasks and Technologies: Implications for Employment and Earnings, "in *Handbook of Labor Economics*, Volume 4, Amsterdam: Elsevier

[2] Arpaia, A. et al. (2009) " Understanding Labour Income Share Dynamics in Europe, " European Economy Economic Papers no.379, European Commission.

[3] Antras, P, (2004) "Is the U.S. Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity," *B.E. Journal of Macroeconomics,* Vol. 4, No. 1, pp. 1-36.

[4] Autor, David, and David Dorn (2013) "The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market" *American Economic Review*, 103(5) pp. 1553-1597.

[5] Autor, David, David Dorn, Lawrence Katz, Christina Patterson, and John Van Reenen (2019)" The Fall of the Labor Share and the Rise of Superstar Firms, "Technical report, National Bureau of Economic Research, *(Quarterly Journal of Economics, Forthcoming)*, `https://economics.mit.edu/files/12979`

[6] Blanchard, Oliver (1997) "The Medium Run," Brookings Papers on Economic Activity, Vol. 2, pp. 89-158.

[7] Chirinko, Robert S. & Debdulal Mallick, (2014). "The Substitution Elasticity, Factor Shares, Long-Run Growth, and the Low-Frequency Panel Model," CESifo Working Paper Series 4895, CESifo.

[8] Chirinko, Robert S., Steven M. Fazzari, and Andrew P. Meyer. (2011) " A New Approach to Estimating Production Function Parameters: The Elusive Capital-Labor Substitution Elasticity. " *Journal of Business and Economic Statistics* 29 (4): 587-94.

[9] Dao, Mai Chi, Mitali Das, Zsoka Koczan, and Weicheng Lian (2017) " Why Is Labor Receiving a Smaller Share of Global Income? Theory and Empirical Evidence, "Technical report, International Monetary Fund.

[10] De Grip, A. and J. Van Loo (2002) "The Economics of Skills Obsolescence: A Review," De Grip, A., J. Van Loo, and K. Mayhew eds. *The Economics of Skilles Obsolescence: Theoretical Innovations and Emripcial Applications*, Elsevier, pp. 1-26.

[11] Elsby et al. (2013) Elsby, M. W. et al. (2013) " The Decline of the US Labor Share, " Brookings Papers on Economic Activity, Fall 2013, pp.1-63.

[12] Galbraith, J. K., (1958) *The Affluent Society*. Boston, Houghton Mifflin.

[13] Grossman et al. (2017) " The Productivity Slowdown and the Declining Labor Share: A Neoclassical Exploration, "NBER Working Paper 23853, Sep. 2017.

[14] Herrendorf, Berthold, Christopher Herrington, and Akos Valentinyi. (2013) " Sectoral Technology and Structural Transformation. " Center for Economic Policy Research Discussion Paper 9386.

[15] Hori, Takeo (2011) "The Effects of Consumption Externalities in an R&D-Based Growth Model with Endogenous Skilled and Unskilled Labor Supply" *Journal of economics* Vol. 102 pp. 29-55

[16] International Monetary Fund (2017) " Understanding the Downward Trend in Labour Income Shares, "Chapter 3, World Economic Outlook, April 2017.

[17] Kaldor, Nicholas (1957). "A Model of Economic Growth". *The Economic Journal.* 67 (268): 591-624. doi:10.2307/2227704. JSTOR 2227704.

[18] Karabarbounis, L. and B. Neiman （2014）" The Global Decline of the Labor Share, "*The Quarterly Journal of Economics*, Vol. 129, No. 1, pp. 61–103, feb, DOI：`http://dx.doi.org/10.1093/qje/qjt032`.

[19] Kuwahara, S. (2006). "Management Ability, Long-run Growth, and Poverty Traps," *Journal of Economics*, 89, pages37–58.

[20] Kuwahara, S. (2019) "Multiplicity and stagnation under the Romer model with increasing returns of R&D" *Economic Modelling* 79, 86–97.

[21] Lawrence, Robert Z. （2015） " Recent Declines in Labor 's Share in US Income：A Preliminary Neoclassical Account, " National Bureau of Economic Research Working Paper 21296, DOI：`http://dx.doi.org/10.3386/w21296`.

[22] Lucas, R. E. Jr. (1988). "On the mechanism of economic development," *Journal of Monetary Economics* 22(1), 3–42.

[23] Neuman, S. and A. Weiss (1995) "On The Effect of Schooling Vintage on Experience-earnings Profiles: Theory and Evidence," *European Economic Review*, 39, pp. 943–945

[24] Oberfield, Ezra, and Devesh Raval. (2014) " Micro Data and Macro Technology. " National Bureau of Economic Research Working Paper 20452.

[25] OECD (2012) " Labour Losing to Capital: What Explains the Declining Labour Share?, " Chapter 3, OECD Employment Outlook 2012.

[26] Ramirez, J. V. (2002) "Age and Schooling Vintage Effects on Earnings Profiles in Switzerland," A. De Grip, J. Van Loo, and K. Mayhew eds. The Economics of Skilles Obsolescence: Theoretical Innovations and Empircial Applications, Elsevier, pp. 83-99

[27] Romer, P. M. (1990). "Endogenous Technological Change," *Journal of Political Economy* 98(5), S71-S102.

[28] Segerstrom, P. (1998) "Endogenous Growth without Scale Effects" *American Economic Review*, 88, issue 5, 1290-1310.

[29] Tran-Nam, B. Truong, C., and Tu, P. (1995). "Human Capital and Economic Growth in an Overlapping-generations Model," *Journal of Economics* 61(2), 147-173.

[30] Vissing-Jorgenson A, Attanasio OP (2003) Stock-market participation, intertemporal substitution, and riskaversion. *American Economic Review Paper & Proceeding* 93(2):383.391

[31] Xu, Shaofeng. 2017 "Volatility risk and economic welfare" "Journal of Economic Dynamics & Control" 80 pp.17-30

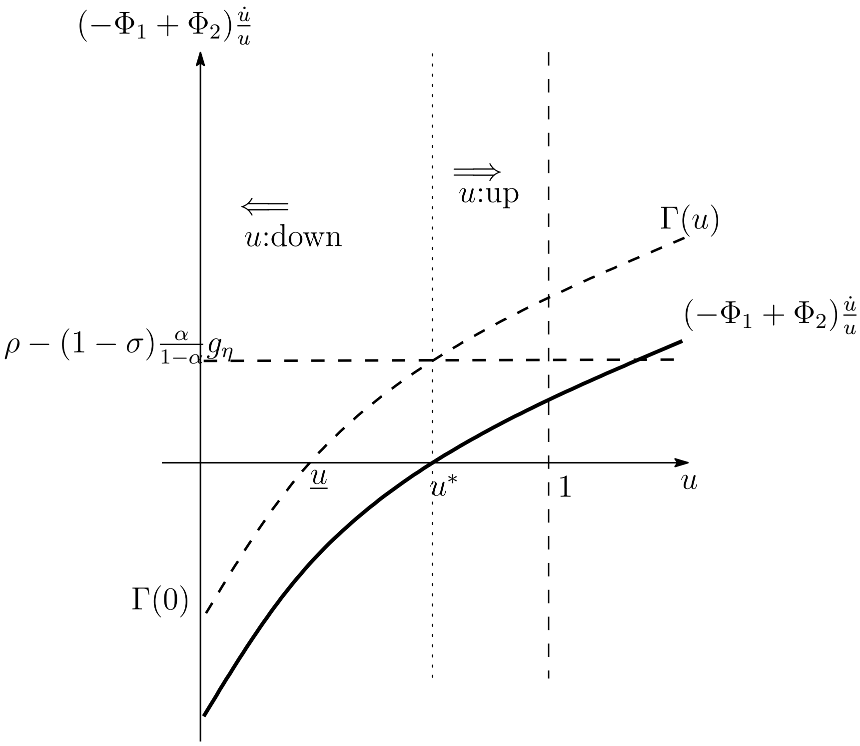

Figure 1: Equilibrium u^* and Dynamics of u

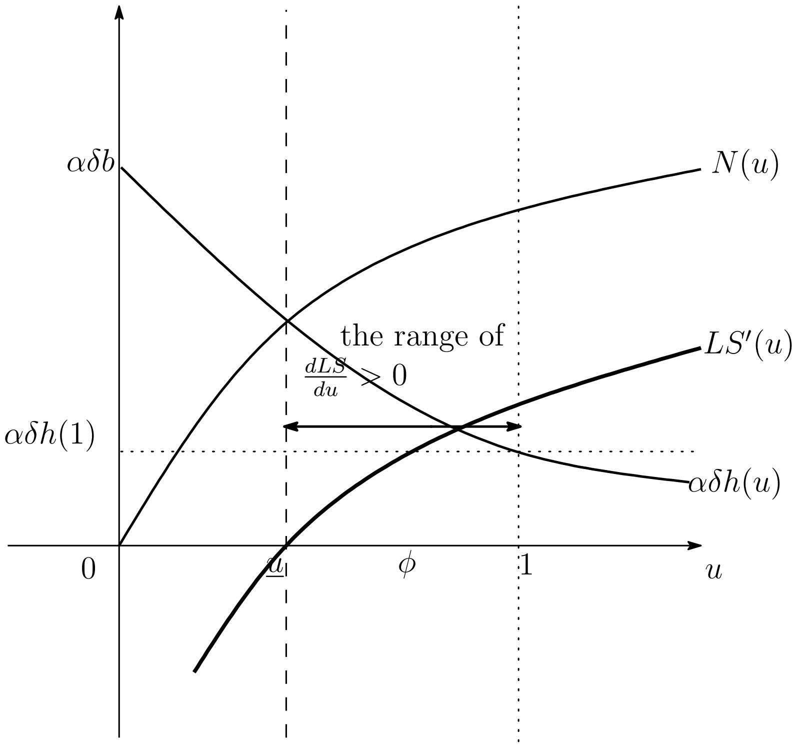

Figure 2: The range of u which derives $dLS/du > 0$