Dynamics of Competitiveness and Multiplicity of Steady States in the Romer Model with Imitations

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Abstract

This study investigates the relationship among the dynamics of market competitiveness, dynamics of economic growth, and long-run growth phase using the Romer (1990) model with expiring patents caused by imitation activities. The main results obtained are as follows: Intense intellectual property rights (IPR; with more difficult imitation) cause the economy to ride on a positive economic growth path with research and development (R&D), and vice versa. In the middle range, the economy might have two steady states: with and without R&D and multiple steady states. In this case, following initial competitiveness, the economy is determined by the converging steady state. Because the arrival rate of imitation indicates its difficulty and can be interpreted to be partially affected by political factors, such as policy parameters, we confirm that the IPR policy can escape the no-growth trap. However, in the middle area, where multiple steady states occur, not only R&D efficiency and imitation intensity but also initial competitiveness determine the converging states.

Keywords: Romer model; imitation; multiple steady states; dynamics of competitiveness.

JEL: E10, E20, O10, O41

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1 Introduction

This study analyzes the relationship between competitiveness and multiple steady states that has not been adequately treated in the literature on endogenous growth. We refer to an important study by Laussel and Nyssen (1999) that analyzed the effects of patent length on multiple steady states. However, their study conducts a partial equilibrium analysis on the labor market. This study clarifies the relationship between imitation and steady states with/without long-run positive growth; the dynamics of market competitiveness; and the effects of innovation, imitation, and the initial statement of competition on the realization of the steady state using the framework of a representative macroeconomic general equilibrium model: the Romer-type research and development (R&D) based growth model.

The Romer (1990) model is related to patent and long-run growth. Because the knowledge has the properties of public goods, non-excludability, and non-rivalness, patents play a central role in economic growth. Moreover, monopolized profit from patents is an incentive for R&D activities, and they drive economic growth. Thus, intellectual property rights (IPR) are the main driving forces of economic growth, and the importance of patents as the main body of IPR has been recognized, with many studies being conducted.

One characteristic of the Romer (1990) model is the perpetual patent. The infinite patent length of the perpetual patent is, of course, unrealistic. Nevertheless, patent length has been variously studied since the 1960s, for example, by Nordhaus (1969) and Judd (1985). Some researchers, such as Iwaisako and Futagami (2003), Futagami and Iwaisako (2007), Lin (2015), and Lin and Shampine (2014), assumed a finite patent length in the endogenous growth model. However, as Lin and Shampine (2014) stated, patent length does not essentially matter with respect to the dynamic properties of endogenous growth (although it would matter in regard to its qualitative properties).

In this study, we relax the assumption of infinite length patents by adding another simple assumption, imitating activities. In this process, we simplify the Romer model by omitting capital (durable goods). Then, as usual, R&D activities create a new intermediate good and a patent is granted to the created knowledge ("blueprint" of the intermediate goods), which yields a sequence of monopolizing profits. In this monopolized sector, we introduce imitation activities with a constant, exogenously given, imitation arrival rate. Then, the monopolized intermediate goods sectors wherein imitations occur become competitive thereafter.

Thus, we obtain the coexistence of monopolizing and competitive sectors, which are analyzed in Matsuyama (1999), Iwaisako and Futagami (2003), and

Kuwahara (2007). These previous studies are different from this study as they considered a fixed-length patent. However, in our model, the monopolistic power expires stochastically. This stochasticity implies that the source of the monopoly power in this study could be interpreted as not simply newly-developed knowledge so that its usage is completely protected by patent, but knowledge with broader meaning, including brand image, competitive location, and so on.[1] Moreover, our objective differs from those of Matsuyama (1999), Iwaisako and Futagami (2003), and Kuwahara (2007). Matsuyama (1999) focused on the fluctuation between monopolized and competitive sectors, Iwaisako and Futagami (2003) focused on welfare and patent length, and Kuwahara (2007) focused on middle-income traps. By contrast, this study focuses on the possibility of multiple steady states yielded by the relationship between innovation and imitation as well as the dynamics of economy and market competitiveness.

Many studies such as Helpman (1993), Lai (1998a, 1998b), and Tanaka and Iwaisako (2014), among others, examine the relationship between patents and imitation using the north-south model. In the north-south model, which is a two-country model with both advanced and developing countries, we can refer to, as a representative arrangement, the model wherein innovation is usually executed in the advanced country and imitation is executed in the developing country. As the model can represent a simple dynamic system with abundant implications, many north-south models have been developed. On another front, the closed-economy model has received little attention with some exceptions such as Kwan and Lai (2003) and Furukawa (2007). This study does consider a closed-economy setting. This is because some developing countries, such as China, have already been growing with R&D while facing the inadequacy of IPR. Moreover, in advanced economies with huge R&D-based high-technology enterprises having monopolistic power, the precautions against monopoly power have been gradually gaining momentum. These phenomena, we think, indicate that the interaction between R&D and imitation activities in both advanced and developing countries' national economies is also important.

This study obtains the following results: We use a Romer model without

[1] Hori (2009) and Hori and Kuwahara (2009) are similar to this study, and we can refer to them for models including broad knowledge, where a new intermediate good firm obtains monopoly power not only through patents but also unique advantages, for example, excellent design and location of plants. Although Hori (2009) and Hori and Kuwahara (2009) assume that an intermediate good supplied by the firm becomes extinct after an intrinsic cost shock to the firm, this study assumes that a monopolist firm loses its monopoly power after the entry of a competitor; therefore, innovative intermediate goods, or the concept of the brand or efficient site location, do not become extinct.

capital accumulation, which does not include transition dynamics against the given market competitiveness. Herein, the newly developed goods market is monopolized, and imitation activities have been destroying the monopolizing power of existing sectors step by step. Thus, dynamics of market competitiveness can be extracted and the competitiveness of the economy is gradually changed, which is the transition dynamic in this economy. In this process, the dynamics are driven by innovation, which increases the monopolized sector, and imitation, which increases the competitive sector, and in the steady states, both numbers are equated, or all sectors become either monopolized or competitive.

When the imitation rate is too high, the profitability of R&D is violated and R&D ceases and the incremental growth of sectors stops, and all sectors become competitive in the long run. In the case with positive R&D, if the mechanism that equates innovative sectors and imitative sectors exists, then the steady states with a constant rate of monopolized and competitive sectors exist, and we can clarify the existence of this type of steady state. However, our setting in this study cannot yield an economy where all sectors are monopolized because we assume an exogenously-given constant imitation rate, which prevents all sectors from being monopolized. Alternatively, we obtain the possibility of multiple steady states. In our model, the long-run steady state with positive R&D is conditioned by two conditions. The first is the no R&D (poverty trap) equilibrium, which is linear. The other gives an inner (real) quadratic curve solution. The domain that satisfies both the poverty trap and real solution conditions, gives multiple steady states. Thus, an economy could have two possible steady states for the given deep parameters, and the selection of the steady state that the economy converges on depends on the competitiveness. Thus, in this study, the selection of the steady state depends not on expectations but on history as suggested by Krugman (1991).

Furthermore, this study assumes stochastic monopoly power, which implies that the imitation rate does not simply reflect the patent length, but rather something of the institutional competitiveness. Then, it is natural that this reflects, at least partially, the IPR protection intensity. In this mechanism, we can consider policy interventions. A lower imitation rate basically provides steady states with R&D, but the existence of history-dependent selection and the quadratic property condition for multiple steady states, means that the policy effects would be less visible.

The remainder of this paper is organized as follows. The model is set up and solved for static equilibria in Section 2. The two types of steady states are derived in Section 3. In Section 4, several topics of economic growth and development are discussed. Section 5 concludes the study.

2 The Model

2.1 Production

This study follows the production structure of the Romer (1990) model. There are three sectors in the present analysis: final goods, intermediate goods, and R&D. Production takes place using two production factors: labor and intermediate goods. Time is continuous, and the price of final goods is normalized to 1.

Final goods are supplied competitively. These goods are produced with labor and a cluster of intermediate goods and are used for consumption (C) and intermediate goods production.[2] Thus, the final goods production function is specified as

$$Y = L_Y^{1-\alpha} \int_0^A \tilde{X}(i)^\alpha di, \quad \alpha \in (0,1), \tag{1}$$

where Y, L_Y, A, and $\tilde{X}(i)$ are the final goods output, labor, number of intermediate goods, and quantile of the ith intermediate good input $i \in [0, A]$, respectively.

From, the assumption of perfect competition in the final goods market, we derive the first-order condition of final goods production as follows:

$$\frac{\partial Y}{\partial L_Y} = (1-\alpha)\frac{Y}{L_Y} = w, \quad \text{and} \quad \frac{\partial Y}{\partial \tilde{X}(i)} = L_Y^{1-\alpha}\tilde{X}(i)^{\alpha-1} = p(i), \tag{2}$$

where w and $p(i)$ are the real wage rate and price of the ith intermediate good, respectively.

The original Romer (1990) model assumed that intermediate goods are produced by final goods, not durable goods, that is, capital. This arrangement is used by, for example, Barro and Sala-i-Martin (1995, Ch6), where every intermediate good is produced by using an $\eta\,(\geq 1)$ unit of final goods. Thus, we omit the capital, making our analysis, which contains other dynamics of market competitiveness, easier.

The final goods are also used as consumption goods and durable goods, namely capital. Thus, the resource constraint of final goods is given as $Y = C + X$, where C and $X \equiv \int_0^A \tilde{X}(i)di$ are consumption and aggregate demand of the final goods from the intermediate goods sector, respectively. The profit from producing the ith intermediate good is

[2] The quality of the cluster, that is, the productivity of intermediate goods, can be regarded as knowledge in this economy.

$$\tilde{\Pi}(i) = p(i)\tilde{X}(i) - \eta\tilde{X}(i). \tag{3}$$

There are patented intermediate goods and non-patented goods at each point of time. The sectors producing the former and latter are the monopolized and competitive sectors, respectively. In the competitive sector, all firms operate freely with the technology; therefore, the goods are competitively supplied, which results in a price equal to the marginal cost. Therefore, the following conditions are obtained:

$$\text{For} \quad i \in \mathcal{C} \;\Rightarrow\quad \tilde{X}_c = \tilde{X}_c(i) = \left(\frac{\alpha}{\eta}\right)^{\frac{1}{1-\alpha}} L_Y, \quad p_c = p_c(i) = \eta, \tag{4}$$

where $\tilde{X}_c = \tilde{X}_c(i)$ and $p_c = p_c(i)$ are derived from the symmetric equilibrium of intermediate goods. In a monopolized sector, a monopolistic firm, or a firm holding a patent, maximizes its profit by considering price as a control variable. Therefore, the first-order condition of the monopolistic firm in the ith sector with the M_ith quality yields

$$\text{For} \quad i \in \mathcal{M} \Rightarrow \quad \tilde{X}_m = \tilde{X}_m(i) = \left(\frac{\alpha^2}{\eta}\right)^{\frac{1}{1-\alpha}} L_Y, \quad p_m = p_m(i) = \frac{\eta}{\alpha}, \tag{5}$$

where $\tilde{X}_m = \tilde{X}_m(i)$ and $p_m = p_m(i)$ are derived from the symmetric equilibrium. Eqs. (4) and (5) imply that each sector shows symmetry between firms in the same sector. Therefore, by denoting s as the share of the competitive sector, sA and $(1-s)A$ represent the number of goods produced in the competitive and monopolized sectors, respectively. From (4) and (5), we obtain

$$X = \left(\frac{\alpha^2}{\eta}\right)^{\frac{1}{1-\alpha}} A\Phi(s;1)L_Y, \tag{6}$$

where $\Phi(s;z) \equiv (1-s) + \alpha^{-\frac{z}{1-\alpha}}s$ $(z = \alpha, 1)$. With respect to Φ, we have $\Phi(0;z) = 1$ and $\Phi(1;z) = \alpha^{-\frac{z}{1-\alpha}} > 1$, and Φ is a linear increasing function of s. Thus, in this version, economic output can increase if the monopolized sector becomes competitive.

Substituting Eqs. (4) and (5) into (1), we obtain the aggregate final food output Y as

$$Y = \left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}} A\Phi(s;\alpha)L_Y. \tag{7}$$

Thus, the changes in Y obey the changes in technology level A, competitiveness c, and labor allocation L_Y. From Eqs. (3), (5), and (7), we have

$$\tilde{\Pi}_m = \left(\frac{1}{\alpha} - 1\right) \eta \tilde{X}_m = \alpha(1-\alpha)\frac{Y}{A\Phi(s;\alpha)}. \tag{8}$$

In Eq. (7), $\Phi(s)$ denotes the change in output between the competitive ($s = 1$) and monopolized ($s = 0$) cases for a given endowment.[3]

2.2 R&D: Entry Activities

In this model, innovation, or entry, is assumed to be the discovery of a new design of intermediate goods, or a new profit opportunity that is added to the existing set of intermediate goods (A). Therefore, the new design of intermediate goods can be denoted as $\dot{A}$. Moreover, accordingly, patents of the designs bear the stream of monopoly profits. The present value of the stream is the value of R&D, or new entry, which we denote as $\tilde{V}$. The dynamics of $\tilde{V}$ are given as

$$r\tilde{V} = \dot{\tilde{V}} + \tilde{\Pi}_m - \mu\tilde{V}, \tag{9}$$

where μ is the depreciation rate of the monopolistic power. The depreciation is caused by competing new firms that gain marginal and instantaneous monopoly profits. Following Grossman and Helpman (1991), among others, we assume that the imitation is exogenous and can be variable through the policy for economic competition.

Because the R&D or new entry activities are assumed to be competitive, the value of firm v is equated to the price of a design or new entry. In this process, it is assumed that R&D firms pay their labor (L_A) wage w and freely use the entire knowledge stock captured by the number of intermediate goods (A). To eliminate the scale effects, we introduce negative effects from population L, which is assumed to be linearly affected. Thus, new intermediate goods are invented in the R&D firm according to the technology:

$$\dot{A} = \delta\, A\, L_A, \tag{10}$$

where $\delta(> 0)$ denotes the R&D or entry efficiency parameter. Hereafter, we call δ, R&D efficiency.

Defining the human capital input of R&D firm j as L_A^j, we assume that the share of the jth firm's innovation share against aggregate innovation $\dot{A}$ is a proposal for the human capital share employed by firm j against the

[3]Of course, endowment is affected by competitiveness.

aggregate employment in the R&D sector: L_A^j/L_A. Therefore, the optimizing problem of the R&D firm is given as

$$\max_{L_A^j} \frac{L_A^j}{L_A} \dot{A}\tilde{V} - wL_A^j (\equiv \pi_R).$$

The free entry of R&D yields $\pi_R = 0$. Therefore, the value of R&D is given as

$$\tilde{V}\dot{A} \leq wL_A, \quad \text{for} \quad L_A \geq 0. \tag{11}$$

Substituting Eq. (10) into (11), we obtain the following condition of positive R&D activity:

$$\tilde{V}\delta A = w \quad \text{or} \quad \tilde{V} = \frac{w}{\delta A}. \tag{12}$$

If inequality $v < \frac{w}{\delta A}$ holds, this implies that R&D activity is not executed. Combining Eqs. (2), (7), and (12), we have the value of v as follows:

$$\tilde{V} = \frac{(1-\alpha)}{\delta}\left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}} \Phi(s;\alpha). \tag{13}$$

3 Dynamics and Steady States

This section analyzes the dynamic properties of the economy and describes the properties of its steady states.

3.1 Dynamics

First, we derive the dynamics of Y. In this study, labor can be used for final goods production (L_Y) or R&D activities (L_A). The market-clearing condition for labor imposes $L = L_Y + L_A$, where L is the total amount of labor in the economy. Hereafter, we define $u \equiv L_Y/L$, wherein $L_Y = uL$ and $L_A = (1-u)L$ are used for simple notation.

From Eqs. (7) and (10), we have

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{u}(t)}{u(t)} + \delta(1-u(t))L + \underbrace{\frac{\alpha^{-\frac{\alpha}{1-\alpha}} - 1}{\Phi(s(t);\alpha)}}_{\equiv \Gamma_Y(s(t))} \dot{s}(t). \tag{14}$$

Because $\alpha^{-\frac{\alpha}{1-\alpha}} > 1$ for $\alpha \in (0,1)$, the coefficient of $\dot{s}(t)$ is positive.

Next, we analyze the case with positive R&D wherein Eq. (9) holds. Thus, by combining Eqs. (8), (9), and (13), we obtain relationship r and $\dot{s}$

as follows:

$$r(t) = \Gamma_Y(s(t))\dot{s}(t) + \frac{\alpha\delta u(t)L}{\Phi(s(t);\alpha)} - \mu. \tag{15}$$

Then, the model is closed by specifying the household. We assume a representative household and that each household inelastically supplies a unit of labor. From this, we identify labor supply at the level of the total population. Furthermore, we assume that the population scale is constant and exogenously given for simplicity. Thus, aggregate labor supply L also denotes the constant population scale in this economy. Consumption is specified as being of the following constant relative risk aversion type: $U = \int_0^\infty e^{-\rho t} logC(t)dt$, where C denotes consumption. Following Romer (1990), we assume that labor is constant, and so the aggregate value and per capita value can be identified. This optimization problem yields the following ordinary Euler equation as an optimizing condition: $\frac{\dot{C}(t)}{C(t)} = r(t) - \rho$. In this study, final goods can be used for intermediate goods production ($\tilde{X}$) or consumption (C). The market-clearing condition for the final goods imposes $Y = \tilde{X} + C$. Moreover, $\tilde{X}$ and Y are respectively derived in Eqs. (6) and (7). Thus, we obtain the relationship between C and Y as follows:

$$C(t) = (1 - c(t))\,Y(t), \text{where} \quad c(t) = c(s(t)) \equiv \frac{\alpha^2}{\eta}\frac{\Phi(s(t);1)}{\Phi(s(t);\alpha)}, \tag{16}$$

and $1 - c$ denotes the marginal propensity to consume. Through a brief calculation, we obtain the following results: $c(0) = \frac{\alpha^2}{\eta}$, $c(1) = \frac{\alpha}{\eta}(< 1)$, and $c'(s) > 0$ for $s \in [0, 1]$; therefore, the condition for positive consumption $c(s) \in (0, 1)$ can be confirmed to be satisfied. Eq. (16) implies that c is a function of competitive index s (and time through s). Then, the dynamics of consumption C are subject to the dynamics of Y and s.

Differentiating Eq. (16) based on time, we obtain

$$\frac{\dot{C}(t)}{C(t)} = \frac{\dot{Y}(t)}{Y(t)} - \underbrace{\frac{\dot{c}(t)}{1 - c(t)}}_{\equiv\tilde{\Gamma}_c(t)}, \tag{17}$$

and

$$\tilde{\Gamma}_c(t) = \underbrace{\frac{\alpha^2}{\eta}\frac{(1-\alpha)\alpha^{-\frac{1}{1-\alpha}}}{\big\{1 - c(s(t))\big\}\Phi(s(t);\alpha)^2}}_{\equiv\Gamma_c(s)(>0)}\dot{s}(t).$$

From the Euler equation $\frac{\dot{C}(t)}{C(t)} = r(t) - \rho$, (14), and (17), we obtain the relation r and dynamics of u and s as follows:

$$r(t) - \rho = \delta(1 - u(t))L + \frac{\dot{u}(t)}{u(t)} + \Gamma_Y\big(s(t)\big)\dot{s}(t) - \Gamma_c\big(s(t)\big)\dot{s}(t). \tag{18}$$

By eliminating $r - \Gamma_Y \dot{s}$ using Eqs. (15) and (18), we obtain the dynamics of u as follows:

$$\frac{\dot{u}(t)}{u(t)} = \frac{\alpha\,\delta\,u(t)\,L}{\Phi(s(t);\alpha)} - \mu - \rho + \Gamma_c\big(s(t)\big)\dot{s}(t) - \delta(1 - u(t))L. \tag{19}$$

Because $s(t)$ indicates the competitive sector share, $\dot{s}(t)$ can be positive or negative when the economy becomes more competitive or monopolizing, respectively. Other terms are always positive. Therefore, we need to discuss the dynamics of varieties and their types of competitiveness captured by s.

To derive the dynamics of s, we define the number of monopolized sectors $M \equiv (1 - s)A$, which implies that

$$\frac{\dot{M}}{M} = \frac{-\dot{s}}{1 - s} + \frac{\dot{A}}{A}. \tag{20}$$

The increment of M is generated by the innovation of a new variety of goods, which is given as $\dot{A} = \delta A L_A$. The decrement of M occurs because of the imitation, which is assumed to be $\mu(1 - s)A$. Therefore, we have

$$\dot{M} = \delta A(1 - u)L - \mu(1 - s)A. \tag{21}$$

By eliminating M from (20) and (21), we have the dynamics of s as follows:

$$\dot{s}(t) = \big(1 - s(t)\big)\mu - s(t)\delta(1 - u(t))L, \tag{22}$$

where it should be noted that the dynamics of s depend on variable s; therefore, the dynamics of u must first be discussed to obtain the dynamics of s. However, we can depict the dynamics of s for u in Fig. 1, wherein we find that s is increasing for all $u \in (0, 1)$.

From Eqs. (19) and (22), we obtain the dynamics of u as follows:

$$\frac{\dot{u}(t)}{u(t)} = \underbrace{\left[\frac{\alpha}{\Phi(s(t);\alpha)} + 1 + s(t)\Gamma_c\big(s(t)\big)\right]}_{A_1(s)} \delta\,u(t)\,L - \underbrace{\left[\Gamma_c\big(s(t)\big)\left\{s(t)\delta L - \big(1 - s(t)\big)\mu\right\} + \mu + \rho + \delta L\right]}_{A_2(s)}. \tag{23}$$

$A_1 > 0$ always holds, but because A_2 contains the term $\delta L - \big(1 - s(t)\big)\mu$, the sign of A_2 is ambiguous. If $A_2 < 0$,[4] then $\dot{u}/u > 0$ for $\forall u \in (0, 1)$, only the long-run feasible equilibrium is $u = 1$, and the economy is caught in a no-growth trap. Because we are interested in the possibility of long-run positive growth, we confine our analysis to the case of $A_2 > 0$.

Using A_1 and $A_2 > 0$, we obtain the phase diagram depicted in Fig. 2, which implies that equilibrium, denoted as $\bar{u}(s)$, exists uniquely, and the dynamics of u are unstable. Because u is a jumpable variable, jumping to $\bar{u}(s)$ and staying there, economic behaviors are derived by rational expectations.

For $\bar{u}(s)$ to be feasible, $\bar{u}(s) \in (0, 1)$ must be satisfied. $\bar{u}(s) \in (0, 1)$ is equivalent to $\frac{\dot{u}}{u}\big|_{u=1} > 0$, namely $A_1\delta L - A_2 > 0$. This condition is converted into $\delta > \bar{\delta}(s)$, where $\bar{\delta}(s) \equiv \frac{\Phi(s;\alpha)}{\alpha L}\big\{\rho + \mu - (1 - s)\mu\Gamma_c(s)\big\}$. When condition $\delta > \bar{\delta}(s)$ is satisfied, the phase diagram shows that an economy with a rational expectation selects unique equilibrium u^* for a given s. It should be noted that allocation rate s varies with time, and so $\bar{u}$ does not immediately have the steady state value of u.

From (23), we can derive the equilibrium labor allocation rate $\bar{u}$ as follows:

$$\bar{u}(s) = \frac{\mu + \rho + \delta L + \Gamma_c(s)\left\{s\delta L - \big(1 - s\big)\mu\right\}}{\left[\frac{\alpha}{\Phi(s;\alpha)} + 1 + s\Gamma_c(s)\right].\delta\, L}. \tag{24}$$

This study utilizes the Romer model without capital accumulation, which has only one stock variable: knowledge. Moreover, the labor allocation between production and R&D and the final goods allocation between consumption and production are uniquely determined against the given knowledge stock A, which fixes economic growth rate g_A. Thus, the usual Romer model without capital contains no transition dynamics, and the economy always stays in a steady state. However, the economy in this study contains the dynamics of the loss of monopolistic power stemming from imitation, which generate transition dynamics. Because the change in s is considered exogenous for all decentralized agents, it cannot be handled but affects the condition through a change in $\Phi(s; z)$. The dynamics of s are discussed in the next section on steady states.

3.2 Steady States

In this section, we derive some conditions for steady states.

[4]For $A_2 < 0$, $\delta < \frac{\mu\left\{\Gamma_c(s)(1-s)-1\right\}-\rho}{\left\{\Gamma_c(s)s+1\right\}L}$ is necessary under $\Gamma_c(s)(1 - s) > 1$ and $\mu > \frac{\rho}{\Gamma_c(s)(1-s)-1}$. Namely, a sufficiently low R&D efficiency and sufficiently high imitation rate would derive $A_2 < 0$.

Combining s and u, we have the following possible steady states:

$$\begin{aligned}
&\dot{s} > 0 (s \to 1) \,\text{and}\, u > 0 \Longrightarrow \{s^*, \bar{u}(s^*)\} = \{1, \bar{u}(1)\} &&: \text{Case I}\,(\mathcal{C}),\\
&\dot{s} < 0 (s \to 0) \,\text{and}\, u > 0 \Longrightarrow \{s^*, \bar{u}(s^*)\} = \{0, \bar{u}(0)\} &&: \text{Case II}\,(\mathcal{M}),\\
&\dot{s} = 0 \,\text{and}\, u > 0 \Longrightarrow \{s^*, \bar{u}(s^*)\} = \{s^*, \bar{u}(s^*)\},\\
&\qquad \text{where } s^* \equiv arg\{s \,|\, (1-s)\mu = s\delta(1-\bar{u}(s))L\} &&: \text{Case III}\,(\mathcal{D}),\\
&\dot{s} > 0 (s \to 1) \,\text{and}\, u = 0 \Longrightarrow \{s^*, \bar{u}(s^*)\} = \{1, 0\} &&: \text{Case I}\,(\mathcal{P}),
\end{aligned}$$

where $\mathcal{C}, \mathcal{M}$, and $\mathcal{D}$, respectively, stem from the initials of competitive, monopolized, and dual. All sectors in the economy in $\mathcal{P}$ are competitive, and those in $\mathcal{M}$ are monopolized. However, in the economy in $\mathcal{D}$, both competitive and monopolized sectors co-exist.

Then, we derive the necessary conditions in the steady state between u and s, which are derived from R&D activities and consumption optimization.

We denote the index steady state values as $*$ and consider our analysis in the inner solution case of u^*, namely $u^* \in (0, 1)$.

Eq. (12) implies $g_v = g_w - g_A$, where $g_Z \equiv \frac{\dot{Z}}{Z}$, namely g_Z, denotes the growth rate of variable Z. Eqs. (2) and (7) imply $g_w = g_Y - g_{L_Y}$. In the steady state, we have $g_w^* = g_Y^* = g_A^*$; thus, we obtain $g_v^* = 0$. From Eqs. (2), (8), (7), and (12), we have $\frac{\pi_m}{v} = \frac{\alpha\delta\bar{u}L}{\Phi(s^*;\alpha)}$. Substituting these results into (9), we have

$$r^* = \frac{\alpha\delta\bar{u}(s^*)L}{\Phi\big(s^*;\alpha\big)} - \mu. \tag{25}$$

Under the steady state, Eq. (16) and the Euler equation give

$$\rho + g_A^* = r^*. \tag{26}$$

From Eqs. (7), (10), and (16), and using u, we conclude that

$$g_Y^* = g_C^* \equiv g^* = g_A^* = \delta(1 - \bar{u}(s^*))L \tag{27}$$

must hold in the steady states.

From Eqs. (25), (26), and (27), we have

$$\bar{u}^*(s) = \frac{\rho + \mu + \delta L}{\left[\frac{\alpha}{\Phi(s^*;\alpha)} + 1\right]\delta L}, \tag{28}$$

where we can easily obtain $\bar{u}^{*\prime}(s) > 0$; namely, $\bar{u}(s)$ is increasing in $s \in (0, 1)$. It should be noted that Eq. (28) is obtained by inserting $\Gamma_c = 0$ into (24). This does not mean that $\Gamma_c = 0$ in steady states. Nevertheless, the effects of Γ_c vanish in the steady states because Γ_c is the coefficient of $\dot{s}$, that is, 0, on the steady states.

From (28), we can derive the following properties on the steady states:

(i) Condition for the existence of an inner solution $\bar{u}^*(1) < 1$, namely, the reverse condition of (29), yields a unique inner solution of type $\mathcal{D}$, $\mathcal{M}$, or $\mathcal{C}$. Because a discussion on stability is necessary, we discuss this later.

(ii) Condition for the existence of a no-growth steady state $\mathcal{P}$ is derived by the condition of $\bar{u}^*(1) > 1$. The economy without R&D ($u = 1$) can arrive at $\mathcal{P}$ by $\dot{s}|_{u=1} > 0$. Thus, $\bar{u}^*(1) > 1$ is the condition of the emergence of a steady state with a poverty trap.

The conditions in (i) and (ii) are transformed into

$$\bar{u}^*(1) \left\{ \begin{array}{c} > \\ < \end{array} \right\} 1 \iff \alpha\delta L \left\{ \begin{array}{c} < \\ > \end{array} \right\} \Omega_0 \quad \text{or} \quad \mu \left\{ \begin{array}{c} > \\ < \end{array} \right\} \alpha^{\frac{1}{1-\alpha}} \delta L - \rho,$$
$$\iff \left\{ \begin{array}{c} \text{A unique steady state related with an inner solution,} \\ \text{At least one steady state related with a corner solution,} \end{array} \right. \tag{29}$$

where $\Omega_0 \equiv (\Lambda+1)(\rho+\mu)\big)$ and $\Lambda \equiv \alpha^{-\frac{\alpha}{1-\alpha}} - 1 (> \alpha > 0)$. Because δL denotes the potential maximum R&D power, a sufficiently small R&D efficiency or excessively high imitation rate causes long-run no growth, and vice versa.

3.3 Steady States and Dynamics of s

The above discussions only imply the possibility of the existence of steady states. In this section, we analyze the dynamic properties and feasibility of converging the steady states. From the above discussions, we derive two equations that condition the steady states and dynamics of s, namely, $\dot{s} = 0$ and $u = \bar{u}^*$. Under the restriction of positive R&D activities, namely $u < 1$, u is uniquely given by Eq. (28). Moreover, Eq. (22) yields the dynamics of s, and the two equations (28) and $\dot{s} = 0$ derived from (22) yield the equilibrium of u and s in the steady state.

The dynamics of this economy are depicted by the dynamics of s, and they can be derived by substituting Eq. (24) into Eq. (22) as follows:

$$\dot{s}(t) = \frac{(1-s)\mu\big\{\alpha + \Phi(s;\alpha)\big\} - s\big\{\alpha\delta L - \Phi(s;\alpha)(\mu+\rho)\big\}}{\alpha + \Phi(s;\alpha)\big\{1 + s\Gamma_c(s)\big\}}. \tag{30}$$

Because we obtain the dynamics of s, we can investigate the possibility of an emerging type of steady state derived in subsection 3.2.

On $\mathcal{C}$, we obtain the necessary condition $\dot{s} > 0$ for $s \to 1$. From this and (30), we have

$$\lim_{s\to 1} \dot{s} = \frac{-\{\alpha\delta L - \Phi(1;\alpha)(\mu+\rho)\}}{\alpha + \Phi(1;\alpha)\{1+\Gamma_c(1)\}} < 0. \tag{31}$$

Thus, the type $\mathcal{C}$ is impossible.

On $\mathcal{M}$, we obtain the necessary condition $\dot{s} < 0$ for $s \to 0$. From this and (30), we have

$$\lim_{s\to 0} \dot{s} = \frac{\mu\{\alpha + \Phi(0;\alpha)\}}{\alpha + \Phi(0;\alpha)} > 0. \tag{32}$$

Thus, type $\mathcal{M}$ is also impossible. However, this condition also shows that $\mathcal{P}$ is possible.

These two impossibilities can be confirmed graphically. From the two equations (28) and $\dot{s} = 0$ derived from (22), we obtain the equilibrium of u and s in the steady state as two panels of Fig. 3. They contain only $\mathcal{D}$ and $\mathcal{P}$ and neither $\mathcal{C}$ nor $\mathcal{M}$.

For emerging $\mathcal{D}$, we obtain the two patterns depicted in Fig. 3, where panel (a) depicts the case of $\bar{u}^*(1) < 1$ and panel (b) shows the case of $\bar{u}^* > 1$. For $\bar{u}^* > 1$, steady state $\mathcal{M}$ vanishes if $u = \bar{u}^*(s)$ and $\dot{s} = 0$ do not intersect with each other. It should be noted that $\bar{u}^*$ is not the transition path of u or s; it is only one of the conditions that determine the steady state. Moreover, Fig. 3 also shows that types $\mathcal{C}$ and $\mathcal{M}$ are impossible.

To derive the condition of $\mathcal{D}$, we need further investigation. From (30), we define the following:

$$\begin{aligned}\Sigma(s) \equiv &(1-s)\mu\{\alpha + \Phi(s;\alpha)\} - s\{\alpha\delta L - \Phi(s;\alpha)(\mu+\rho)\},\\ &= \Lambda\rho s^2 + s\{\Omega_1 - \alpha\delta L\} + (1+\alpha)\mu,\end{aligned} \tag{33}$$

where $\Lambda(\alpha) \equiv \alpha^{-\frac{\alpha}{1-\alpha}} - 1$ and $\Omega_1(\mu;\alpha,\rho) \equiv \mu(\Lambda - \alpha) + \rho$. We can easily conform $\Lambda > \alpha (> 0)$; therefore, $\Omega_1 > 0$. Then, $s^* = \arg\{s|\dot{s} = 0\}$ is given by $s = \arg\{s|\Sigma(s) = 0\}$. s^* gives the long-run equilibrium market share of the competitive firm. Because $\Sigma(s)$ is a quadratic function of s, the equation $\Sigma(s) = 0$ is a quadratic equation; therefore, we have two roots at most and no roots at the least. Combining this and $\Sigma(0) = \mu(1+\alpha) > 0$ (for $\mu > 0$), we have three possible cases, and combining the dynamic properties, we have a three-phase diagram: panels (a), (b), and (c) in Fig. 4, where $\tilde{s}$ is the value of s that gives the extremum of function Σ. Σ is defined as

$$\tilde{s} = \arg\{s|\Sigma'(s) = 0\}.$$

In Fig 4(b), we obtain three points of s that give multiple steady states, $\mathcal{D}$, $\mathcal{D}'$ and $\mathcal{P}$. $\mathcal{D}'$ is unstable and it can be realized at measure 0. When $s \in (\mathcal{D}, \mathcal{D}')$, s is converging $\mathcal{D}$, and when $s \in (\mathcal{D}', \mathcal{P})$, s is converging $\mathcal{P}$. Thus, we have the following lemma:

Lemma *In the case of Fig 4(b), multiple steady states merge, and the initial s determines the converging steady state against the given parameters such as μ and δL.*

It should be noted that the condition in (29) is given by $\Sigma(s)$ as follows:

$$\bar{u}^*(1) \left\{ \begin{matrix} > \\ < \end{matrix} \right\} 1 \iff \Sigma(1) \left\{ \begin{matrix} < \\ > \end{matrix} \right\} 0 \iff \alpha\, \delta\, L \left\{ \begin{matrix} > \\ < \end{matrix} \right\} \Omega_0, \tag{34}$$

where $\Omega_0 = (\Lambda + 1)(\rho + \mu)$. Thus, $\Sigma(1) > 0$ yields panels (b) and (c) in Fig. 4, where $\mathcal{P}$ always emerges. It should be noted that the condition obtained in (34) is depicted as a line on $(\mu, \alpha\delta L)$-plain.

Result 1 $\Sigma(1) > 0$ *always yields the poverty trap, and* $\Sigma(1) < 0$ *always yields a unique steady state $\mathcal{D}$ with an inner solution. The threshold values of* $\Sigma(1) = 0$ *yield* $\alpha\delta L = \Omega_0(\mu; \rho, \alpha)$*. Moreover, higher ρ and μ, lower δ and L, less durable time attitude, less protective IPR, lower R&D efficacy, and less R&D input endowment cause the emergence of $\mathcal{P}$, and vice versa.*

The possibility is considered wherein steady state $\mathcal{D}$ exists or not under the condition of $\Sigma(1) > 0$, namely, under the existence of steady state $\mathcal{P}$. That is, if $\mathcal{D}$ exists, multiple equilibria emerge. Because $\Sigma(s) = 0$ is a quadratic equation, to generate $\mathcal{D}$, we have additional conditions $\Sigma'(0) < 0$, $\Sigma'(1) > 0$, and $D_\Sigma > 0$ besides $\Sigma(1) > 0$, where D_Σ denotes the discriminant of the equation $\Sigma(s) = 0$.[5].

Eq. (33), $\Sigma'(0) < 0$, and $\Sigma'(1) > 0$ (and $\alpha\delta L < \Omega_0$) immediately give the following condition:

$$\Omega_1 < \alpha\delta L < \min[\Omega_1 + 2\Lambda\rho,\, \Omega_0], \tag{35}$$

where $\Omega_1 \equiv \mu(\Lambda - \alpha) + \rho$. Because $\Lambda > \alpha$ and $\Omega_1 < \Omega_0$ can be obtained immediately, an intermediate goods efficiency (α) adjusted maximum R&D ability (δL) that satisfies this interval always exists.

[5]Discriminant $D_\Sigma > 0$ indicates the existence of root(s), and $\Sigma'(0) < 0$ and $\Sigma'(1) > 0$ give $\tilde{s} \in (0, 1)$.

Solving Eq. (35) on μ, we obtain

$$\max\left[\frac{\alpha\delta L-\rho(1+2\Lambda)}{\Lambda-\alpha},\frac{\alpha\delta L-(\Lambda+1)\rho}{\Lambda+1}\right]<\mu<\frac{\alpha\delta L-\rho}{\Lambda-\alpha}. \tag{36}$$

Until now, we obtain the result that we have the possibility of multiple steady states if the economy satisfies condition (35). To draw a conclusion, we must check the last condition $D_\Sigma>0$, which is given as follows:

$$D_\Sigma(\mu)=\big(\Omega_1(\mu)-\alpha\delta L\big)^2-4\Lambda\rho(1+\alpha)\mu>0.$$

Thus, we also obtain a (quadratic) relationship between μ and $\alpha\delta L$. Because we consider the case of $\alpha\delta L>\Omega_1$, the above inequality becomes

$$\alpha\delta L>\Omega_1(\mu)+2\sqrt{\Lambda\rho(1+\alpha)\mu}. \tag{37}$$

It should be noted that because this constraint is the sum of a linear function ($\Omega_1(\mu)$) and a functional square root of a linear function ($2\sqrt{\Lambda\rho(1+\alpha)\mu}$), it is a type of a quadratic curve. For $\mu>0$, equation $\alpha\delta L=\Omega_1+2\Lambda\rho$ yielded from (35) and equation $\alpha\delta L=\Omega_1(\mu)+2\sqrt{\Lambda\rho(1+\alpha)\mu}$ yielded from (37) have a unique intersection at $\mu=\frac{\Lambda\rho}{1+\alpha}$. Thus, combining (35) and (37), we obtain the emergence condition of the steady state on the $(\mu,\alpha\delta L)$-plain, as depicted in Fig. 5.

This figure and Lemma provide the main results of this study, as follows:

Result 2 *A higher μ and lower adjusted R&D ability $\alpha\delta L$ basically cause a poverty trap. Thus, an IPR protecting policy stimulates long-run growth and vice versa. However, between the domains with only $\mathcal{D}$ and only $\mathcal{P}$, we obtain the domain in which both $\mathcal{D}$ and $\mathcal{P}$ exist. In this area, there exists the domain with only $\mathcal{P}$ as in above, and so a simple increasing $\alpha\delta L$ or decreasing μ policy might fail the economy in riding on the transition path converging to D. Furthermore, the selection of the converging steady states in the area with $\mathcal{D}$ and $\mathcal{P}$ depends on initial competitiveness s. Thus, in this study, the converging point is determined historically and not by expectations.*

Thus, the mechanism of selecting a steady state from two possibilities in this study is "history" and not "expectations" (Krugman, 1991). Our study also contains the political factor μ, which might change the result—that is, history can be changed by policies.

Furthermore, we assume a stochastic monopoly power. These arrangements imply that μ reflects not only patent length, but also some institutional competitiveness. Then, it is natural that this competitiveness could be, at least partially, considered to be affected by the attitude of the government, μ can be considered a policy variable in this study. Fig. 5 shows that we have two policy instruments in this study: one is the potential maximum R&D ability (δL), and the other is IPR protection which affects the imitation rate (μ).

This study assumes stochastic monopoly power, which implies that the imitation rate does not reflect a simple patent length, but rather some institutional competitiveness. Then, it is natural that this reflects, at least partially, some IPR protection intensity. In this mechanism, we can consider the policy intervention. Because more intense IPR (policy for making imitation more difficult) tends to ensure that the economy rides on a positive economic growth path with R&D and vice versa, a policy that restricts imitation basically provides steady states with R&D. However, in the middle range, the economy might have two steady states; those with and without R&D. Furthermore, the presence of history-dependent selection and the quadratic property condition for multiple steady states, would make the policy effects less visible. To ensure that the economy remains in a unique positive long-run growth state, the government must attempt to move the economy into the $\{\mathcal{D}\}$ domain.

4 Conclusion

This study analyzes the relationships among R&D-based growth, discrimination, and the dynamics of economy and market competitiveness. The economy in this study has long-run steady states with and without R&D, or long-run R&D-based growth (denoted as $\mathcal{D}$) and a no-growth trap (denoted as $\mathcal{P}$ in this study). They respectively correspond to an economy that is both monopolized and competitive and one that is only competitive. An economy with a high imitation rate and low R&D ability is stuck in the steady state without R&D, and vice versa. In addition, an economy with parameters with middle values has both types of steady states, and the economic convergence is a no-growth trap if it has sufficiently high competitiveness in the initial period, and vice versa.

Because this study assumes that monopolizing power expires stochastically, the source of total factor productivity growth can be interpreted as not merely R&D, but something that yields inherent advantages for firms while being legally less protected than when holding a patent. Meanwhile, under the assumption that intermediate goods are produced using final goods, a more competitive market structure means that more intermediate goods are supplied competitively. This implies that the instantaneous production level is increased, but the driving force of long-run growth is weakened. Thus, a policy that enhances excessive entry of competitors for short-run increments to output might decrease the long-run growth of output, which depends positively on the implicit knowledge of such an entry in the niche market or construction of an optimal supply network, which cannot be protected by a patent.

This study is preliminary, and several points still need to be addressed. First, it lacks the micro-foundation of discrimination, which occurs stochastically. The fundamental results depend on the exogenously given arrival rate of imitation, which is constant but presumed to be affected by policy. Furthermore, although the present version assumes a myopic government, theoretically, optimal growth should be solved to analyze a rational growth policy. We believe this is possible.

Second, the imitation or competitor's entry probability is exogenously given and constant. That is, both old and new monopolistic firms confront their competitors with the same probability. This also stems from the assumption that the cost of imitation or the competitor's entry is zero. Existing studies on imitation assume an imitation cost. To include the optimal activities of a competitor would enrich the analysis through comparison with the existing model with imitation.

Third, because we use the Romer (1990) model, each sector is either already developed or not. Once a sector once becomes competitive, it is never monopolized by, for example, a new monopolist who develops higher quality goods. This property clearly weakens the direction of the model for monopolizing.

These points constitute important future agendas in the stream proposed by this research.

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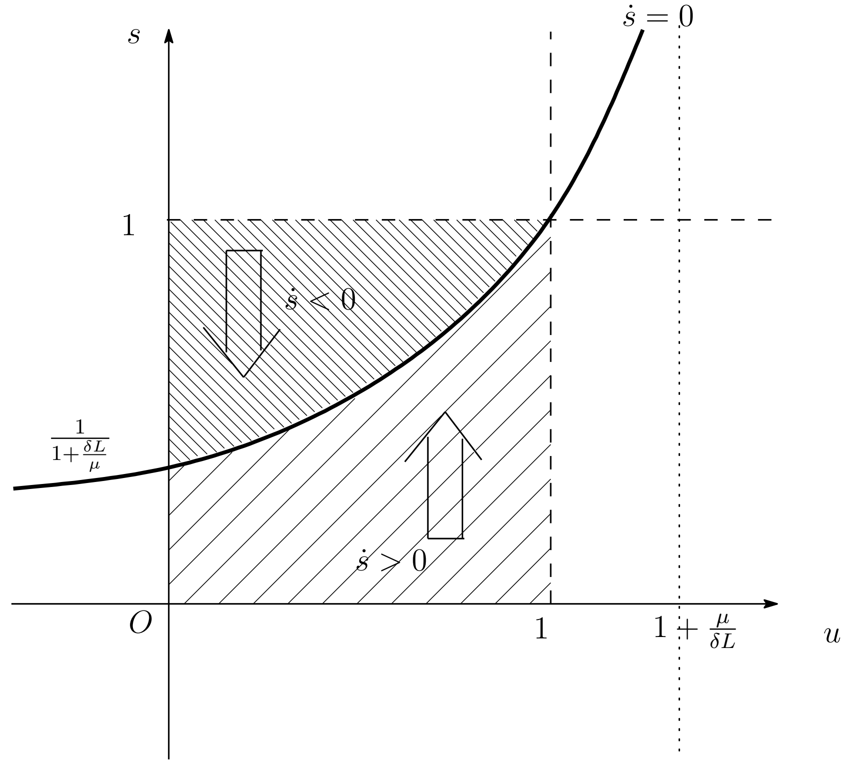


Figure 1: Dynamics of s on the u-s plain

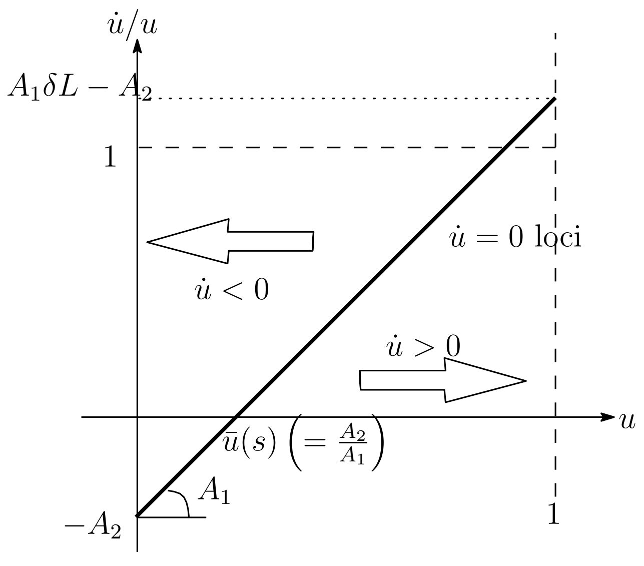


Figure 2: Dynamics of u

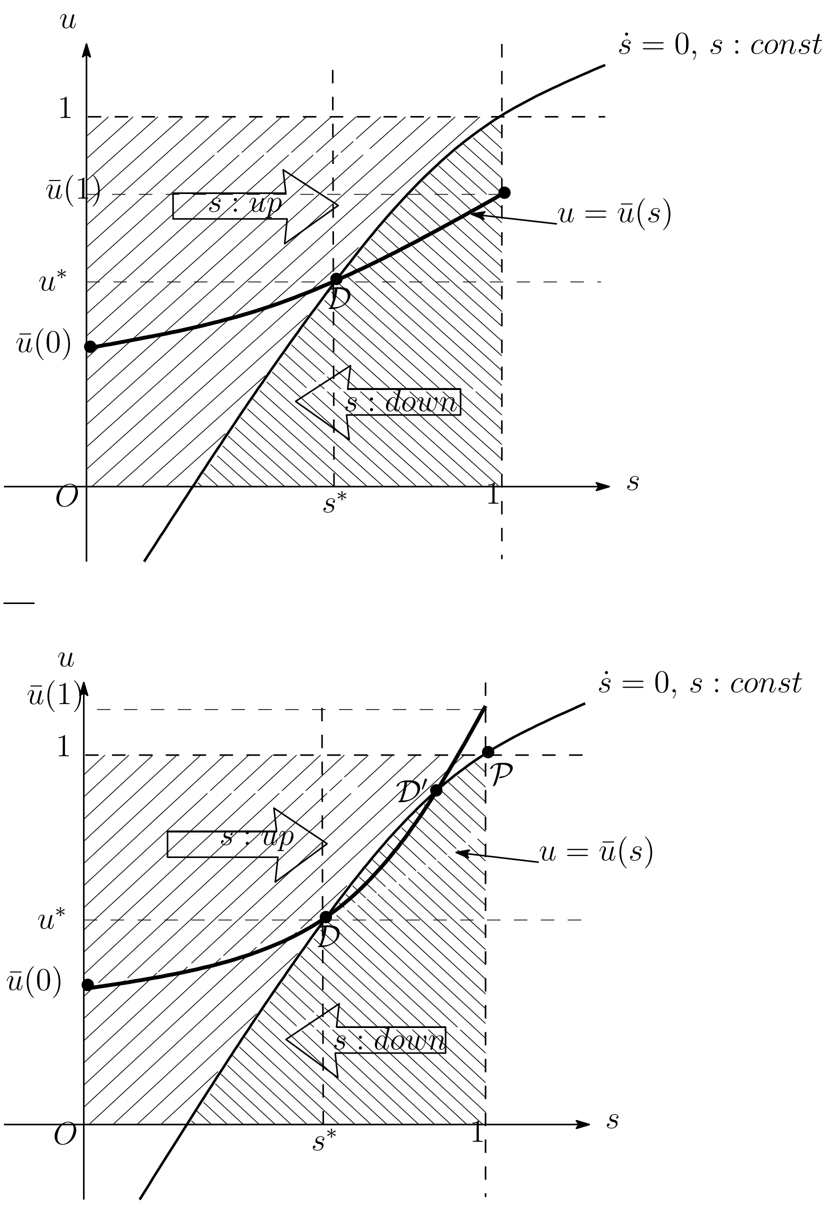
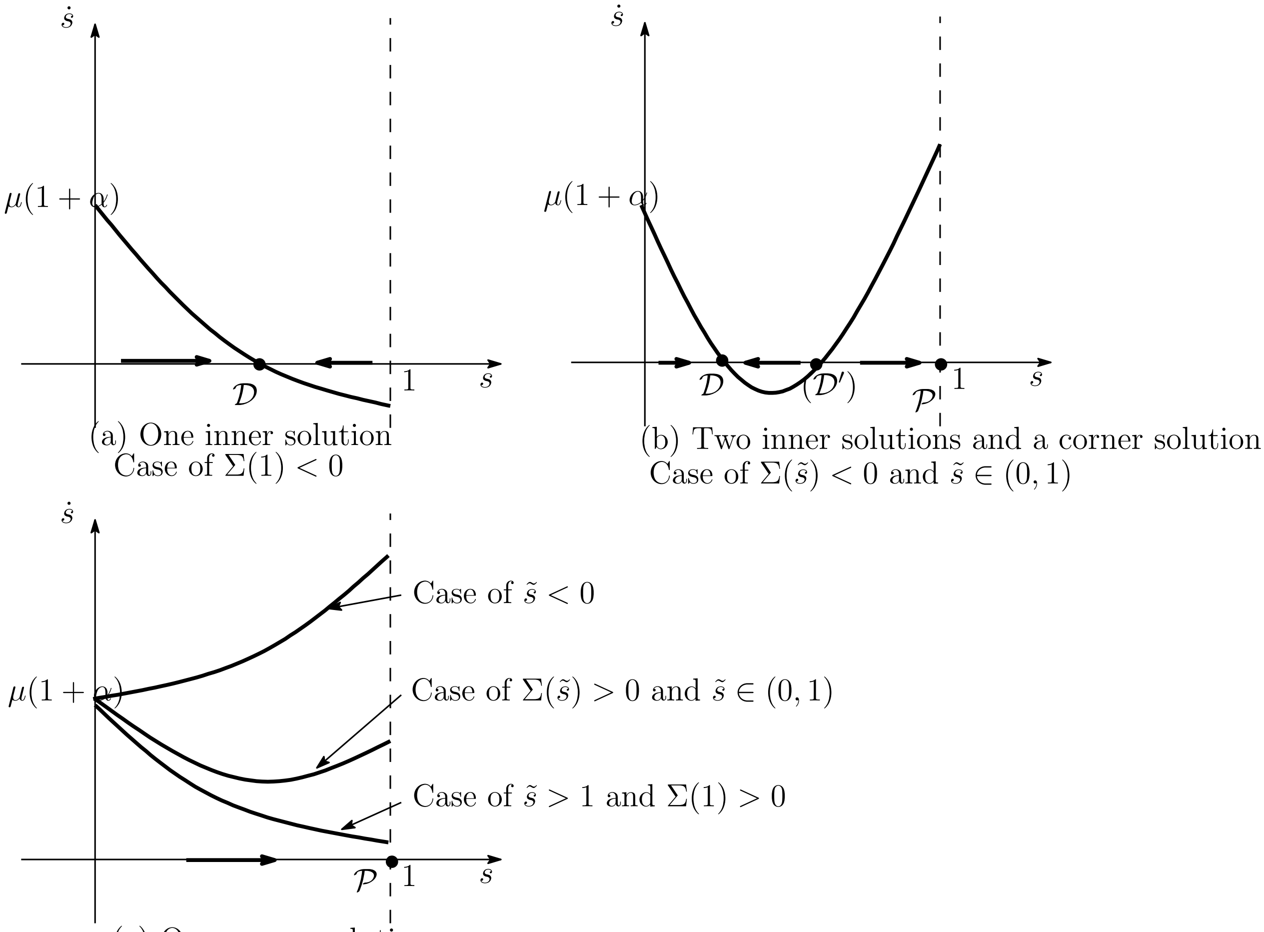


Figure 3: Steady state on the $(s-u)$ plain



(a) One inner solution
Case of $\Sigma(1) < 0$

(b) Two inner solutions and a corner solution
Case of $\Sigma(\tilde{s}) < 0$ and $\tilde{s} \in (0,1)$

(c) One corner solution

Figure 4: Dynamics of s

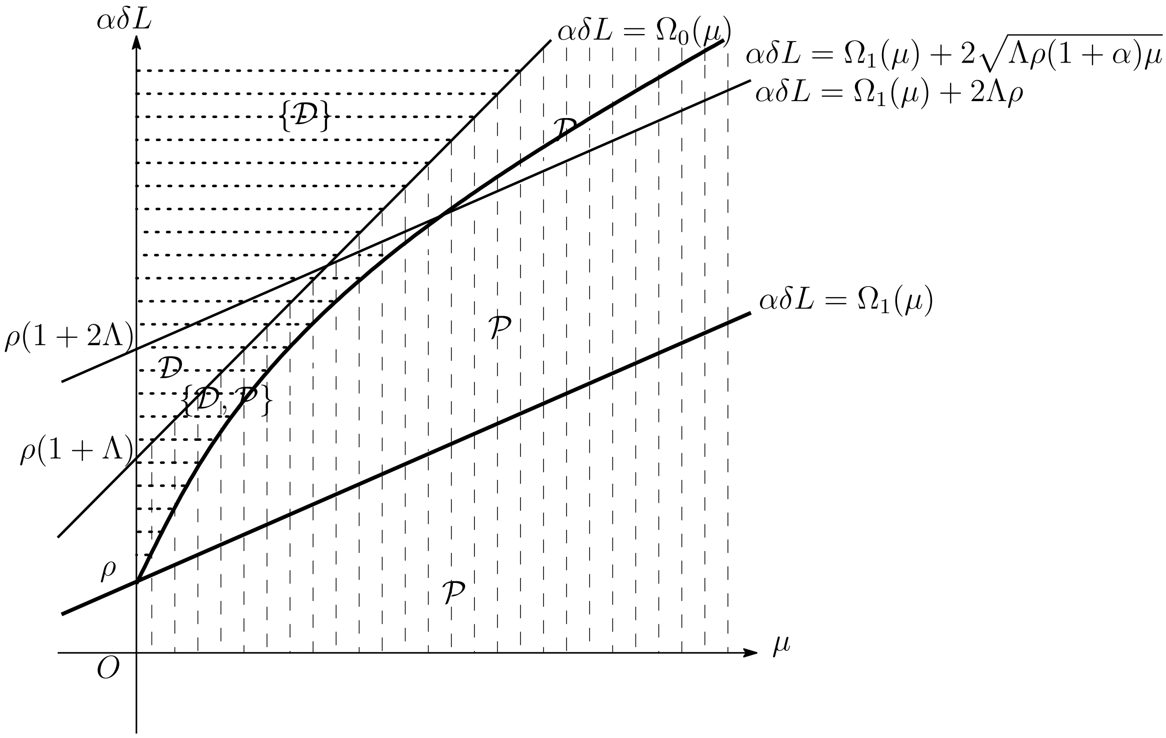


Figure 5: Emergence of steady states