The Fall of the Labor Share and Possibility of the Autor and Dorn Model*

Shiro Kuwahara[1]

Abstract

For the analyze of the fall of the labor share in the world, this paper surveys several empirical studies pertaining to the reduction of labor share in modern economies. It then uses the Autor and Dorn (2013) model, which depicts the change of labor allocation between goods-producing and service sectors. Among others, we investigate how the model works and to what extent it can replicate the fall of the labor share, and show some possible direction of extensions of the Autor and Dorn model by introducing intertemporal decision, among others.

1 Introduction

Although many empirical studies, such as Maddison (1982), have shown the stability of the labor share since post-World War II, we have many others that posit the contrary. These include Blanchard (1997), Karabarbounis and Neiman (2014), Autor et al. (2017), Dao et al. (2017), who revealed that a decline in labor share is experienced by both advanced and developing economies. This is termed job polarization and it seems to have been breaking down the abundant middle class that consists of industrial workers, the core members of the Affluent Society (Galbraith 1958). Furthermore, it is already yielding poverty and various social problems.

In 1970s, the world experienced polarization where the co-existence of growing advanced economies and poverty-stricken economies was surmounted by a substantial economic growth of some developing countries (World Bank 1993). To date, there seems to be another polarization of the world *à la* 21st century, which is the reemergence of the society class that was resolved by the decades of powerful and global economic growth.

On this process of world economy's transition, some scholars such as Bell's (1974) post-Industrialization and Toffler's (1980) "the third wave." recognize the change. They point out the end of the dominance of industrial production and the emergence of the knowledge creation as the main driver of the economy. This is considered the emerging world with knowledge-based growth, also called the information - oriented society characterized by

*This paper greatly owe to the discussion with Tatsuichi Kaneko. Of course, all remaining errors are sole responsibility of the author.

[1]corresponding author, University of Hyogo, E-mail: kuwahara@em.u-hyogo.ac.jp

persistent innovation in the computer industry (Moore's law). However, we have another important change, the tertiary industrialization, famously coined by the Petty-Clark's law (Petty 1690, Clark 1940).

In this paper, we therefore consider the Autor and Dorn (2013) model, which explores the persistent innovation in the computer industry and dynamic allocation of unskilled labor between service- and goods-producing sectors. Elasticity of substitution between capital and labor is one of key concepts of the model. Karabarbounis and Neiman (2014) estimate an elasticity of substitution to be greater than 1, and they conclude that decreasing relative prices of capital goods trigger a shift of an input factor from labor to capital. On the contrary, Chirinko (2008), who surveyed and evaluated a large number of studies that measured this elasticity, estimated that a value of the elasticity ranges from 0.4 to 0.6. [2] The simple way of relating the decreasing price of capital and decreasing labor share could be the elasticity of substitution that is greater than 1; on the contrary, many empirical studies state it should be less than 1.

However, Autor et al.(2019) support our view of the elasticity of substitution that is greater than 1. They used micro panel data from the U.S. Economic Census since 1982 to demonstrate empirical patterns to assess a new interpretation of the fall in the labor share based on the rise of "superstar firms." If globalization or technological changes push sales towards the most productive firms in each industry, the product market will be increasingly concentrated because industries will be dominated by superstar firms, which have high markups and a low value-adding labor share.

Schwellnus et al. (2018) posited that technological changes in the investment of goods-producing sector and a greater global value chain participation have declined the labor shares; however, the effect of the technological changes is less clear. Countries with falling labor shares have experienced both a decline at the technological frontier and takeover of market shares by top advanced firm with low labor shares.

Another important approach on the declining labor share is one by Piketty (2014) who shows that it is the historical principle the property that capital endowment tends to concentrate and broaden inequality. Equality that we have been experiencing after World War II through economic growth is an exception; he uses the key equation $r > g$. However, some economists such as Summers (2014) and Mankiw (2015) refute the claims made by Piketty. Here, we partly note Mankiw's (2015) critique of Piketty. He reinterpreted the key equation $r > g$ into $f'(k) > n$ and insisted that his reinterpretation does not trigger the dynamic inefficiency. According to Mankiw, the earning rate potential is reduced by (1) marginal propensity rate of wealth 3%, (2) population growth 2%, and (3) tax 2%. For instance, wealth accumulation faces a depreciation rate of total 7%, and, according to Mankiw, this may not lead to dystopia as Piketty implied. By following Vissing-Jørgenson and Attanasio (2003) and Xu (2007), we obtain $r = \theta g + \rho + n > g$ by using the CRRA parameter $\theta > 1$ and the usual Euler equation derived from Ramsey model, $\theta g = r - \rho - n$. Therefore, holding $r > g$ is, as Mankiw points out, a usual case, and it is further need to check the results of the world with $r > g$.

Mankiw's critique, however, is not a complete explanation of the declining labor share; thus, we focus on the decreasing labor share by using a model with the elasticity of substitution in this paper. In this literature, many interesting models have been proposed.

[2] Many studies such as Antras (2004), Chirinko, Fazzari, and Meyer (2011), Oberfield and Raval (2014), Chirinko and Mallick (2014), Herrendorf, Chirinko, Fazzari, and Meyer (2011), Herrington, and Valentinyi (2013), Oberfield and Raval (2014), Herrendorf, Herrington, and Valentinyi (2013), and Lawrence (2015) estimate elasticities below 1.

For example, Acemoglu (2003) used the neoclassical structure, which contains multisector tied with constant elasticity of substitution (CES) technology, to show that there are capital-augmenting technological and factor shares changes along the transition path. However, as described by the standard growth models, economy endogenously becomes the labor-augmenting change in a long run. This paper provides microfoundations, where an economic agent endogenously chooses a labor-augmenting technological change. This implies that any distortion or disturbance such as tax policy and changes in labor supply or savings might change factor shares in the short run; however, these distortions or disturbances may have no, or at least little, effect on the long-run factor distribution of income. The necessity of the labor-augmenting innovation for the long-run growth is reinforced by the Uzawa theorem (Uzawa 1961). However, this condition contradicts the observed steadily falling relative price of capital equipment adjusted quality.

Notably, the standard model is insufficient to explain the social problem that triggers polarization into labor and capitalist class. Thus, a model that details the labor structure is essential.

As already indicated, we use the Autor and Dorn (2013) model, where unskilled labor is divided into routine (employed in goods-producing sector) and manual (employed in a service sector). We then describe the transition of labor force from goods-producing sector into the service sector. The model is successfully applied in some aspects of real phenomena. This paper does not focus on the theoretical aspects of the model; thus, some results and important factors are eliminated. In particular, we describe the model and show how to use it extensively in future research.

The paper is organized into 6 sections. Section 2 gives a description of the model; Section 3 derives the asymptotic steady states in the long run; Section 4 describes the market economy of the model and its properties; Section 5 discusses the necessities of future application of the model. Lastly, Section 6 concludes the paper.

2 The description of the model

2.1 Final goods production and Labor Supply

Following Autor and Dorn (2013) model, we assume that the final goods (Y_g) are produced by abstract work (L_a) that is executed by employing skilled labor, and composited intermediate input denoted Z, and Z is made by routine work (L_r) by employing un-skilled labor and computer (X). We also assume that the elasticity of substitution between L_a and Z on Y_g production is 1, and that between L_r and X on Z is $\sigma_r > 1$. Thus, we obtain the following function:

$$Y_g = L_a^{1-\beta}\left\{(\alpha_r L_r)^\mu + (\alpha_x X)^\mu\right\}^{\frac{\beta}{\mu}}, \quad \beta, \mu \in (0,1), \tag{1}$$

where Z is defined as

$$Z \equiv \left\{(\alpha_r L_r)^\mu + (\alpha_X X)^\mu\right\}^{\frac{1}{\mu}}, \quad \mu > 0$$

where $\mu > 0$ denotes $\sigma_r = \frac{1}{1-\mu} > 1$, which captures the property that computer and routine labor are 'gross substitutes'. This means that routine labor is replaced by computer owing to a cheaper computer price (denoted by p_x).

The unskilled labor, denoted by L_U) supplies either *manual* or *routine* labor, and itis inelastically supplied at unit mass ($L_U = 1$).

The skill of the manual labor is homogenous. Therefore, if all worker are employed in the manual labor, the labor supply for manual work (L_m) is unity ($L_m = L_U = 1$).

In the routine work, there is the efficiency denoted by η with the density function $f(\eta)$, and each worker has an inherent efficiency. We assume that the aggregate efficiency labor supply for routine work is also unity, and so we have

$$\int_{\eta \in U} \eta f(\eta) d\eta = 1 \tag{2}$$

where S denotes the set of unskilled labor, and the labor supply of routine labor is given as

$$L_r = \int_{\eta \in U_r} \eta f(\eta) d\eta, \tag{3}$$

where U_r denotes the set of labor that work as routine workers.

L_a is the number of employees in the final goods sector; however, since the routine work contains the efficiency parameter, L_r denotes the efficiency number of employee.

To obtain an analytical solution, we specify the density function. Following Autor and Dorn (2013), we also adopt the exponential distribution:

$$f(\eta) = e^{-\eta}, \quad \text{for } \eta \in [0, \infty]$$

We respectively denote the wage rate offered for a unit of manual and routine labor as w_m and w_r. Each worker with efficiency of η can gain w_m if s/he works as a manual worker, and $w_r \eta$ if s/he works as a routine worker.

Each worker selects the work depending on labor revenue, and the wage rates are given. The worker with $\eta \geq w_m/w_r (\equiv \bar{\eta})$ works as the routine worker, and the worker with $\eta < \bar{\eta}$ works as the manual worker. Thus, $\bar{\eta}$ represents the threshold efficiency that divides the type of work the unskilled worker selects. Thus, (3) is rewritten as

$$L_r = \int_{\bar{\eta}}^{\infty} \eta f(\eta) d\eta. \tag{4}$$

Then, we derive the resource constraint of labor.

$$1 = L_m + \int_{\bar{\eta}}^{\infty} f(\eta) d\eta \tag{5}$$

This equation gives the following expression (see Appendix for detail derivation):

$$L_r = \left\{1 - \log(1 - L_m)\right\}(1 - L_m) (\equiv g(L_m)). \tag{6}$$

2.2 Conditions derived from optimizations

We derive some conditions derived from household and three production sectors: final goods, service, and intermediate goods sector. All production sectors are assumed to be perfectly competitive. Therefore, each sector's price equates to its marginal cost, and no profit is left on a firm.

The Final Goods Sector The profit of the final goods sector π_g is given as

$$\pi_g = Y_g - w_a L_a - w_r L_r - p_x X,$$

where we adopt the final goods as a numéraire, so that the price of the final goods is unity.

The optimal conditions derived by final goods firm are given as follows:

$$\frac{\partial \pi_g}{\partial L_a} = 0 \Longrightarrow \frac{\partial Y_g}{\partial L_a} = w_a \tag{7}$$

$$\frac{\partial \pi_g}{\partial L_m} = 0 \Longrightarrow \frac{\partial Y_g}{\partial L_r} = w_r \tag{8}$$

$$\frac{\partial \pi_g}{\partial X} = 0 \Longrightarrow \frac{\partial Y_g}{\partial X} = p_x \tag{9}$$

The Service Sector The profit of the service sector π_s is given as

$$\pi_s = p_s Y_s - w_m L_m$$

and the production technology is given as

$$Y_s (= C_s) = \alpha_s L_m \tag{10}$$

Constant returns of this production technology yield the zero-profit condition (instead of F.O.C.) as follows:

$$(p_s Y_s =) p_s C_s = w_m L_m \tag{11}$$

and by combining (10) and (11), we have

$$p_s \alpha_s = w_m. \tag{12}$$

This condition implies that the price of the service p_s and the wage of manual labor w_m are tied.

The Intermediate Goods (Computer) Sector The profit of the intermediate goods sector π_x is given as

$$\pi_x = p_x X - Y_x$$

and the production technology is given as

$$X = \delta_0 Y_x e^{\delta t}, \tag{13}$$

where a positive constant δ denotes the exogenously given innovation rate of the intermediate goods that is presumed as a computer. δ_0 (θ in Autor and Dorn 2013) denotes the efficiency parameter, which equals the price of X at initial period $t = 0$ (in Autor and Dorn 2013, the initial period is $t = 1$). Following Autor and Dorn (2013), we set the price of X at initial period to unity, which yields $\bar{\delta} = 1$.

Since this sector is also constant returns to scale, obtained condition is the zero-profit condition:

$$p_x = \frac{Y_x}{X} = e^{-\delta}. \tag{14}$$

Household To close the model, we assume that all consumers/workers have identical CES utility functions over consuming final goods and services:

$$u = (c_g^\rho + c_s^\rho)^{\frac{1}{\rho}}, \quad \rho < 1. \tag{15}$$

The elasticity of substitution in consumption between goods and service is derived as $\sigma = \frac{1}{1-\rho}$.

3 The long-run steady states

Since the model contains no distortion such as monopoly and externality, we can derive the equilibrium conditions by considering a social planner problem because it is a convenient way to get the long-run allocation between L_m and L_r. Normalized population scale makes the identification of per capita and aggregate values possible.

The Planner Problem Since the resource constraint of final goods $Y_g = C_g + Y_x$ and $Y_x = p_x X$ from (14), we have $C_g = Y_g - p_x X$. Thus, given the sequence of p_x, the social planner problem is given as

$$\max_{X, L_m} \left[L_m^{\frac{\sigma-1}{\sigma}} + \left(Y_g - p_x X\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{16}$$

which is subject to (1) and (6). Thus, the basic structure of the Autor and Dorn (2013) model is the continuous sequence of instantaneous decision making, and unique intertemporal factor is the incessantly-decreasing factor price of computer goods. This captures the innovation in this study.

This problem yields the following two conditions:

$$\frac{\partial Y_g}{\partial X} = p_x, \tag{17}$$

$$L_m^{-\frac{1}{\sigma}} = (Y - p_x X)^{-\frac{1}{\sigma}} \frac{\partial Y_g}{\partial Z} \frac{\partial Z}{\partial L_r} \{-\log(1 - L_m)\},. \tag{18}$$

From the discussion given in Appendix A, we obtain the following condition from (18):

$$\frac{1}{\sigma} \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \frac{\beta - \mu}{\beta} \Rightarrow \lim_{X \to \infty} X^{\beta - \mu - \frac{\beta}{\sigma}} = \left\{ \begin{array}{c} \infty \\ 1 \\ 0 \end{array} \right. \Rightarrow L_m = \left\{ \begin{array}{l} 0 \\ L_m^* \in (0,1) \\ 1 \end{array} \right. . \tag{19}$$

Asymptotic Labor Allocation Since the price of computer goods p_x falls to zero asymptotically, (17) implies that computer goods are limited to

$$\lim_{t \to \infty} X(t) = \infty.$$

From the above expression and $L_r \leq 1 << \infty$, we obtain

$$\lim_{t \to \infty} \frac{Z}{\alpha_x X} = 1.$$

Following Autor and Dorn (2013), let $x \sim y$ be a shorthand for the notation that $\lim_{t\to\infty} x/y = 1$, we can obtain the following expressions:

$$Y_g = \underbrace{L_a^{1-\beta}}_{=1} Z^\beta \sim (\alpha_x X)^\beta \tag{20}$$

$$p_x = \frac{\partial Y_g}{\partial X} \sim \beta \alpha_x^\beta X^{\beta-1} \tag{21}$$

Therefore, we get $p_x X \sim \beta(\alpha_x X)^\beta$. Then, substituting equations obtained above, $Y_g \sim (\alpha_x X)^\beta$ and $p_x X \sim \beta(\alpha_x X)^\beta$, and the zero profit condition of computer sector, $p_x X = Y_x$, into the resource constraint of goods ($Y_g = C_g + Y_x$), we have

$$C_g = Y_g - p_x X \sim (\alpha_x X)^\beta - \beta(\alpha_x X)^\beta = \kappa_1 X^\beta = (1-\beta)Y_g, \tag{22}$$

where $\kappa_1 \equiv (1-\beta)\alpha_x^\beta$.

From the resource constraint of final goods, we have $Y_g = Y_x + C_g$, then

$$Y_g = Y_x + C_g \sim X e^{-\delta t} + \kappa_1 X^\beta = (\alpha_x X)^\beta \tag{23}$$

Therefore, we obtain $Y_X = X e^{-\delta t} = (\alpha_X^\beta - \kappa_1) X^\beta = \beta \alpha_x^\beta X^\beta$, which yields

$$\frac{\dot{Y}_X}{Y_X} = \frac{\dot{X}}{X} - \delta = \beta \frac{\dot{X}}{X}, \tag{24}$$

which yields

$$\frac{\dot{X}}{X} = \frac{\delta}{1-\beta}. \tag{25}$$

From (23) and (25), we have

$$\frac{\dot{Y}_g}{Y_g} = \frac{\dot{Y}_x}{Y_x} = \frac{\dot{C}_g}{C_g} = \beta \frac{\dot{X}}{X} = \frac{\beta\,\delta}{1-\beta}. \tag{26}$$

Thus, we obtain the following lemma:

Lemma 1 We have the following results that

$$\lim_{t\to\infty} p_x X = \infty, \quad \lim_{t\to\infty} \frac{p_x X}{Y} = (const) \in (0, \infty).$$

Proof) We obtain $\frac{\dot{p}_x}{p_x} = -\delta$ (from (14)), and $\frac{\dot{X}}{X} = \frac{\delta}{1-\beta} > \delta$, which yield $\frac{\dot{(p_x X)}}{p_x X} = -\frac{\delta\beta}{1-\beta} > 0$. Furthermore, uniting $\frac{\dot{Y}_g}{Y_g} = \frac{\delta\beta}{1-\beta}$, $\frac{p_x X}{Y}$ is a constant positive value less than infinity. (Q.E.D)

4 Description of the market economy of the Autor-Dorn model

Here, we explore the relationship between goods and service. Autor and Dorn (2013) analyze the model by solving the social planner problem, and so the price of service does not display. That said, GDP, sum of market values of goods and services cannot be calculated. In section, we analyze how the market economy works because the derivation of GDP depends on it. Thus, we introduce the price dynamics of the service (p_s). Under perfect competition, we have zero profit condition of the service sector (which yields $p_s\alpha_s = w_m$), arbitrage condition between routine and manual labor (which yields $w_m = w_r$), and marginal principle of routine work (which yields $w_r = \frac{\partial Y_g}{\partial L_r}$); therefore, we obtain the following equation:

$$p_s\alpha_s = w_m = w_r = \frac{\partial Y_g}{\partial L_r} \tag{27}$$

where $w_m = w_r$ holds under $L_m \in (0, 1)$. The utility function we use in this study is given in (15), and this economic agent is a representative household; therefore, the family structure is an epitome of the whole economy's population structure, and so this household has the labor endowment with a distribution function given as $f(\eta) = e^{-\eta}$. We assume this is the distribution of labor efficiency. Since each labor endowment is assumed to be normalized to 1, the aggregate, average, and representative household's values in this study can be denoted the same. Thus, the maximizing problem of the representative household can be given as follows:

$$\max_{c_g, c_s} \quad \left(C_g^{\rho} + C_s^{\rho}\right)^{\frac{1}{\rho}}, \tag{28}$$

$$\text{s.t.} \quad C_g + p_s C_s = w_m L_m + w_r L_r + w_a L_a. \tag{29}$$

This problem yields the following optimal condition:

$$\left(\frac{C_s}{C_g}\right)^{\rho-1} = p_s \tag{30}$$

Then, using $X \sim \infty$, we have

$$\begin{aligned} w_r L_r = \frac{\partial Y_g}{\partial L_r} L_r &= \frac{\beta}{\mu} L_r^{1-\beta} [...]^{\frac{\beta}{\mu}-1} \mu \alpha^{\mu} L_r^{\mu-1} L_r \\ &= \beta \frac{\alpha_r^{\mu} L_r^{\mu}}{\alpha_r^{\mu} L_r^{\mu} + \alpha_x^{\mu} X^{\mu}} Y_g \sim (a_x X)^{\beta-\mu}. \end{aligned} \tag{31}$$

This yields $\frac{w_r L_r}{Y_g} = (\alpha_x X)^{-\mu} \sim 0$, which implies that the routine share in goods production converges to 0 (infinitely small).

From (22), we have the following equation:

$$C_g = (1-\beta) Y_g. \tag{32}$$

This implies that in the long run, the consumption C_g is converging to the constant rate of Y_g, consumption propensity (C_g/Y_g) depends on β, and not on σ and μ, and then goods consumption (C_g) is growing infinitely. The product, and namely consumption, of the service is given by ($Y_s =$)$C_s = \alpha_m L_m$, and the constant long-run value of L_m yields $C_s(= Y_s)$, because (unskilled) labor endowment is assumed to be constant. Therefore, $p_s \sim \infty$ is realized for $\forall \rho \in (\infty, 1)$, given in Table 1. Incessant innovation of computer makes incessant price down of computer and it realizes the incessant goods' price down. In this study, goods' price is taken as numéraire, the incessant goods' price down is represented by incessant service price up. This price change disturbs the balance of share between goods and services, and this imbalance is adjusted through the shift of unskilled labor between routine and manual labor.

ρ	$-\infty$	$\cdots$	0	$\cdots$	1
σ_u	0	$\cdots$	1	$\cdots$	∞
$p_s \sim$			∞		

Table 1: Long-run value of p_s

ρ	$-\infty$	$\cdots$	0	$\cdots$	1
σ_u	0	$\cdots$	1	$\cdots$	∞
		$\frac{\dot{p_s}}{p_s} > \frac{\dot{Y_g}}{Y_g}$	|	$\frac{\dot{p_s}}{p_s} < \frac{\dot{Y_g}}{Y_g}$	
$Y \sim$		$p_s Y_s$	|	Y_g	
$\dot{Y}_g/Y_g \sim$		$\frac{(1-\rho)\beta\delta}{1-\beta}$	|	$\frac{\beta\delta}{1-\beta}$	

Table 2: Long-run value of Y_g and $\dot{Y}_g/Y_g$

GDP Owing to the introduction of prices, we analyzed the whole economic activity level, that is, the GDP. In this regard, GDP is defined as $Y \equiv Y_g + p_s Y_s$, because these expressions are the whole production in this economy. From (11), (30) and (32), we have

$$p_s = \left(\frac{(1-\beta)Y_g}{\alpha_m L_m} \right)^{1-\rho}. \tag{33}$$

When $L_m \neq 0$ holds, namely, under $\forall \rho < 0$ and $\exists \rho \in (0,1)$, the growth of L_m stops in the long run; therefore, (30) and (32) yields

$$\frac{\dot{p_s}}{p_s} = (1-\rho)\frac{\dot{Y_g}}{Y_g}.$$

Combining the above equation and (26), we have the following condition:

$$\frac{\dot{p_s}}{p_s} \sim \frac{\beta\delta}{1-\beta}(1-\rho) \left\{ \begin{array}{c} > \\ < \end{array} \right\} \frac{\dot{Y_g}}{Y_g}, \quad \text{for} \quad \rho \left\{ \begin{array}{c} \leq 0 \\ \in (0,1) \end{array} \right. . \tag{34}$$

This condition immediately gives

$$Y \sim \left\{ \begin{array}{c} p_s Y_s \\ Y_g \end{array} \right. , \quad \text{for} \quad \rho \left\{ \begin{array}{c} \leq 0 \\ \in (0,1) \end{array} \right. . \tag{35}$$

Regarding $\rho < 0$, the long-run main sector, a sector that produces the most added value, becomes the service sector. Thus, ρ, namely, elasticity of substitution on consumption $\sigma_u = 1/(1-\rho)$, determines the main (most valued) sector. Furthermore, the growth rate of the service-oriented economy is higher than that of goods-production economy.

Labor allocation and Distribution Next, we analyze the labor allocation and distribution on labor.

The threshold on more valued production, whether services or goods, is $\rho = 0$ namely $\sigma = 1$, but the threshold on labor allocation between manual and routine is $1/\bar{\sigma} = \frac{\beta-\mu}{\beta}(< 1)$, which is balanced on $\bar{\sigma}(> \sigma)$ (See Appendix 3.3).

ρ	$-\infty$	$\cdots$	0	$\cdots\cdots$	1
σ_u	0	$\cdots$	1	$\cdots\lvert\bar{\sigma}\rvert\cdots$	∞
$L_m \sim$		1		$\lvert L_m^*\rvert$	0
$Y \sim$		p_sY_s	$\vert$	Y_g	

Table 3: Long-run value of Y_s under $\beta > \mu$

ρ	$-\infty$	$\cdots$	0	$\cdots\cdots$	1
σ_u	0	$\cdots$	1	$\cdots$	∞
$L_m \sim$			1		
$Y_s \sim$		p_sY_s	$\vert$	Y_g	

Table 4: Long-run value of Y_s under $\beta < \mu$

In the case of $\beta > \mu$, $\bar{\sigma} > 1$, we have the following values: In the case of $\beta < \mu$, $\bar{\sigma} < 0$, we have the following values:

Therefore, only sufficiently low μ could yield the asymptotic steady state with $L_m \sim 0$, and combining it with the sufficiently large σ yields the steady state.

However, the global properties depend on ρ, and the economy with $\rho < 0$ yields service-weighted economy and the one with $\rho > 0$ yields goods-weighted economy. These expressions are derived from the utility but from the production structure, where good production is diverse, but the service production has the upper bound caused by the given production factor endowment.

On the analysis of Autor and Dorn (2013), because skilled labor works in the goods sector and their income is assumed to be maintained, they assume $\rho > 0$. As a result, GDP, the aggregate added value, mainly depends on the production of goods, and so this arrangement cannot depict the "tertiary industrialization". Furthermore, when we use the word "tertiary industrialization", we often regard the service sector as the main sector of the economy; However, in the real world, the most advanced sectors are the financial sectors or ITC sectors that provide service. The service sector of the present model is, at most, a retail business that mainly employs the unskilled labor as salespeople. Introducing an advanced service sector, such as the financial sector, would be necessary to cover the tertiary industrialization for the important theme.

5 Discussions

From the above discussions, we confirm the following features of the Autor and Dorn (2013) model: (i) long-run phase is determined by the utility parameter, and under $\rho > 1$, the long-run main engine is fixed at the goods production sector; (ii) under the assumption of (i), the long-run labor allocation is determined by the goods productive; (iii) the model essentially lacks the intertemporal decision-making, which makes the meaning of the interest rate and that of the capital share ambiguous. Future research can be based on the abovementioned properties. It can extend from the Autor and Dorn (2013).

First of all, we should introduce the population growth. Many results stem from the magnitude of the given labor endowment and increasing final goods derived by the computer goods with increasing efficiency. Thus, the introduced population growth would give the bifurcation of the main production between goods and services by the parameter of elasticity of substitution between goods and services.

Endogenous labor supply should be important. We would therefore live in the wealthy world, with higher wages realized through innovation, and we might enjoy more leisure time by reducing labor supply (about 3 hours a day, which is the famous prediction by Keynes 1930). Decreasing trends of labor time are observed in the many countries in the long run (Jones 2016). By contrast, De Vries (1994) insists that the positive change of attitude for labor supply and consumption laid out the groundwork for the industrial revolution, and he calls it "industrious revolution," and this idea is moderilized by Hobara and Kuwahara (2020). In the US, the most advanced capitalist economy, people work harder, even at the sacrifice of leisure time (Schor (1992). However, in the long time span, we observe the decreasing trends of labor time in many countries (Jones 2016), which implies that the increasing or decreasing of working hours is an important issue on the economic growth and labor share.

Secondly, and more interesting from the view of the economic growth theory, the intertemporal decision-making should be introduced. In Autor and Dorn (2013) model, computer goods are regarded as capital, but non-durable goods; in actual sense, a computer is a consumable good nowadays. Thus, all decision-making and all goods' durability are *temporal*, and the intertemporal connection is only continuously decreasing the computer price. To avoid this defect, we can change the production function (1) by introducing capital stock K, which refers to durable goods. This capital stock is presumably not depreciating into, for example, the following:

$$Y_g = L_a^{1-\beta-\gamma} K^{\gamma} \{(\alpha_r L_r)^{\mu} + (\alpha_x X)^{\mu}\}^{\frac{\beta}{\mu}}, \quad \beta, \mu \in (0,1). \tag{36}$$

In this case, the resource constraint is made into

$$Y_g = C_g + \dot{K} + p_x X. \tag{37}$$

Thus, the extended Autor and Dorn model with essentially dynamical or intertemporal mechanism should be developed.

Thirdly, since we would be faced with the maximum service economy in the modern economy, the transition from industrial economy to the service economy (tertiary industrialization) would be analyzed. Many studies such as Matsuyama (1999, 2001), Funke and Strulik (2000), Kuwahara (2007, 2013, 2019), Irmen (2005), and Zilibotti(2009) explore the transition from the production-based economy to knowledge-based economy, but the studies that explore the rise of service economy are few. Thus, this direction of extension of the Autor and Dorn (2013) is worthy, but the arrangement of service goods is symmetric in the model. Therefore, service and goods are necessities in the economy. The demand of service is always an inner solution, and the share of expenditure on goods and service is constant. As a result, there is no transition of expenditure from goods to services. For example, the property $\lim_{C_s \to 0} \frac{\partial U}{\partial C_s} = \bar{U_{C_s}} < \infty$ would give the demand factor of emerging service-oriented economy. Combining that with the supply factor that is served in the Autor and Dorn (2013) would yield an effective future research.

Lastly, as seen in Section 4, the production structure of the present study is not encompassing. However, mechanism that is more exhaustive could work in the future labor market. Currently, unskilled labor is the only substituted labor in the routine work; however, innovation of AI might substitute all types of work in future through all type of machinery. Such economy might be depicted as follows:

$$Y_g = (L_a^{\mu_a} + X_a^{\mu_a})^{\frac{1-\beta}{\mu_a}} \left(L_r^{\mu} + X^{\mu}\right)^{\frac{\beta}{\mu}}, \quad Y_s = a_m(L_m^{\mu_m} + X_m^{\mu_m})^{\mu_m}, \quad \dot{p}_{X_a}, \dot{p}_{X_m} < 0,$$

where X_a and X_m respectively denote inputs of computer (like AI) that can substitute the L_a and L_m, and μ_a, μ_r, p_{X_a}, p_{X_m} denote related elasticity of substitution and prices, respectively. As Autor et al. (2019) note, the roles of "superstar" firm have been important, and this implicitly shows that all other labor's post become weak. In spite of the weakening of labor, the present study yields no unemployment, because the labor market is perfect, and therefore, introducing the unemployment might depict the realistic future economy, and derive relevant economic policies.

6 Conclusion

This article surveys the empirical studies with regard to the decreasing labor share, and uses theoretical background derived from the Autor and Dorn (2013). Despite the result of the elasticity of substitution of capital being less than 1, the share of capital is increase. This implies the more elaborate model about the resource allocation of labor and capital and the Autor and Dorn (2013) model has the basic structure that can prove that; however, further studies are needed in this field.

Furthermore, for the purpose of the analyzing the transition of the labor share, the dynamics of the labor supply, as well as capital accumulation, making labor supply endogenous would be effective.

The ultimate goal of the studies includes increasing the welfare of the world. Many economic agents is a labor as well as a citizen, the resolution of the decreasing labor share, which implicitly yields the decreasing labor income and afford for social lives in broad meaning. As Keynes (1930) notes, labor is shorter.

7 Appendix

7.1 The derivation of the function $g(\cdot)$

From the resource constraint of labor, we have

$$1 = L_m + \int_{\bar{\eta}}^{\infty} f(\eta) d\eta. \tag{38}$$

Here, L_r denotes the effective labor input on the routine sector, namely efficiency η times the density of labor with efficiency η, $f(\eta)(= e^{-\eta})$, therefore, we have

$$L_r = \int_{\bar{\eta}}^{\infty} \eta e^{-\eta} d\eta.$$

Calculating this by using the rule, we have

$$\begin{aligned} L_r &= \int_{\bar{\eta}}^{\infty} \eta e^{-\eta} d\eta = \left[-\eta e^{-\eta} - e^{-\eta}\right]_{\bar{\eta}}^{\infty} \\ &= \bar{\eta} e^{-\bar{\eta}} + e^{-\bar{\eta}} - \underbrace{\lim_{\eta\to\infty} \bar{\eta} e^{-\bar{\eta}}}_{\to 0} - \underbrace{\lim_{\eta\to\infty} e^{-\bar{\eta}}}_{\to 0} \\ &= (1+\bar{\eta}) e^{-\bar{\eta}}. \end{aligned} \tag{39}$$

Then, we consider the resource constraint on the unskilled labor. From (5), we have

$$\begin{aligned} L_m &= 1 - \int_{\bar{\eta}}^{\infty} e^{-\eta} d\eta \\ &= 1 - \left[-e^{-\eta}\right]_{\bar{\eta}}^{\infty} = 1 - [-\underbrace{\eta^{-\infty}}_{\to 0} + e^{-\bar{\eta}}] \\ &= 1 - e^{-\bar{\eta}}. \end{aligned} \tag{40}$$

What we want is the relationship between L_r and L_m, and so we eliminate $\bar{\eta}$. For this purpose, we derive the value of $e^{-\bar{\eta}}$ from (39), and so we did that with $\bar{\eta}$ from (40):

$$\log(1 - L_m) = \log e^{-\bar{\eta}} = -\bar{\eta}. \tag{41}$$

From (40) and (41), we respectively obtain $e^{-\bar{\eta}} = 1 - L_m$, and $\bar{\eta} = -\log(1 - L_m)$, and substituting these two equations into (39) for eliminating $\bar{\eta}$, we get the function $g(\cdot)$ as follows:

$$L_r = \left\{1 - \log(1 - L_m)\right\}(1 - L_m)(\equiv g(L_m)). \tag{42}$$

7.2 Derivation of Eq.(19) from Eq.(18)

In the planner problem, we obtain the following conditions:

$$\frac{\partial U}{\partial X} = 0 \Rightarrow \frac{\partial Y_g}{\partial X} = \beta \frac{Y_g}{Z} \alpha_x^{\mu} X^{\mu-1} = p_x (= e^{-\delta t}), \tag{43}$$

$$\frac{\partial U}{\partial L_m} = 0 \Rightarrow L_m^{-\frac{1}{\sigma}} = (Y_g - p_x X)^{-\frac{1}{\sigma}} \frac{\partial Y_g}{\partial L_m}, \tag{44}$$

where we have the following:

$$\frac{\partial Y_g}{\partial L_m} = \frac{\partial Y_g}{\partial Z} \frac{\partial Z}{\partial L_r} g'(L_m) = -\beta L_a^{1-\beta} Z^{\beta-\mu} \alpha^{\mu} L_r^{\mu} \frac{dg(L_m)}{dL_m}. \tag{45}$$

The partial derivatives are made into

$$\frac{\partial Y_g}{\partial Z} = \beta Z^{\beta-1} \tag{46}$$

$$\frac{\partial Z}{\partial L_r} = Z^{1-\mu} \alpha^{\mu} L_r^{\mu}, \tag{47}$$

Here, we have

$$L_a = 1, \tag{48}$$
$$g'(L_m) = \log(1 - L_m) = -\bar{\eta}, \tag{49}$$
$$\int_{\bar{\eta}}^{\infty} e^{-\eta} d\eta = \left[-e^{-\eta}\right]_{\bar{\eta}}^{\infty} = 0 - (-e^{-\bar{\eta}}) = e^{\bar{\eta}}. \tag{50}$$

From (49), we obtain

$$e^{-\bar{\eta}} = 1 - L_m, \quad \text{namely} \quad L_m + e^{-\bar{\eta}} = 1, \tag{51}$$

which is just a resource constraint on unskilled labor.

Here, we impose the conditions on asymptotic steady states as follows:

$$C_g \sim \kappa_1 X^{\beta}, \tag{52}$$
$$Z \sim \alpha^{\mu} X^{\mu}. \tag{53}$$

Substituting above conditions (46), (47) and (48) into (45), we have

$$L_m^{-\frac{1}{\sigma}} = -\kappa_L (X^{\beta})^{-\frac{1}{\sigma}} L_a^{1-\mu} Z^{\beta-\mu} g(L_m)^{\mu-1} g'(L_m) \tag{54}$$

where $\kappa_L \equiv \kappa_1^{-\frac{1}{\sigma}} \alpha_r^{\mu} \beta$, and $g'(L_m) = \log(1 - L_m)(= -\bar{\eta})$.

Thus,

$$\lim_{t\to\infty} \frac{\partial U}{\partial L_m} = 0 \quad \sim \quad \underbrace{L_m^{-\frac{1}{\sigma}} g(L_m)^{1-\mu} g'(L_m)^{-1}}_{\equiv \Lambda(L_m)} = \lim_{X\to\infty} \kappa_L X^{\beta-\mu-\frac{\beta}{\sigma}} \tag{55}$$

Therefore, the limit value of the RHS depending on the exponential parameter $\beta - \mu - \frac{\beta}{\sigma}$. The results are summarized as follows:

$$\beta - \mu - \frac{\beta}{\sigma} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \Leftrightarrow \frac{1}{\sigma} \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \frac{\beta - \mu}{\beta} \Rightarrow \lim_{X\to\infty} X^{\beta-\mu-\frac{\beta}{\sigma}} = \left\{ \begin{array}{c} \infty \\ 1 \\ 0 \end{array} \right. . \tag{56}$$

This condition is derived from the optimal conditions derived from the planner problem, and so L_m must be controlled to satisfy this equation. In decentralized economy, it is also satisfied by the optimizing behavior of agents.

To derive the behavior of L_m, it is necessary to confirm the properties of $\Lambda(\cdot)$. Taking the limit of the function Λ, we have the following expressions:

$$\lim_{L_m\to 0} \Lambda(L_m) = \underbrace{0^{-\frac{1}{\sigma}}}_{\to\infty} \underbrace{\left[\{1 - \underbrace{\log(1-0)}_{\to \log 1 = 0}\}(1-0)\right]^{1-\mu}}_{\to 1} \underbrace{\frac{1}{\log(1-0)}}_{\to\infty} = \infty,$$

$$\lim_{L_m\to 1} \Lambda(L_m) = \underbrace{1^{-\frac{1}{\sigma}}}_{\to 1} \underbrace{\left[\{1 * (1-1) - (1-1)\log(1-1)\}\right]^{1-\mu}}_{\to 0} \underbrace{\frac{1}{\log(1-1)}}_{\to 0} = 0.$$

On the middle item of the RHS, we have

$$\lim_{L_m \to 1} [\{1 - \log(1 - L_m)\}(1 - L_m)] = \lim_{\mathcal{L} \to 0} \mathcal{L} - \mathcal{L} \log \mathcal{L} = 0 - 0 = 0, \quad (57)$$

where we use the following result: $\lim_{\mathcal{L}\to 0} \mathcal{L} \log \mathcal{L} = \lim_{\mathcal{L}\to 0} \frac{\log \mathcal{L}}{1/\mathcal{L}} = \lim_{\mathcal{L}\to 0} \frac{1/\mathcal{L}}{-1/\mathcal{L}^2} = 0$, which is derived by using L'Hospital's Rule. Uniting this and the continuity of $\Lambda(L_m)$, L_m can take $\Lambda(L_m) \in (0, \infty)$, therefore, we have

$$\frac{1}{\sigma} \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \frac{\beta - \mu}{\beta} \Rightarrow \lim_{X \to \infty} X^{\beta - \mu - \frac{\beta}{\sigma}} = \left\{ \begin{array}{c} \infty \\ 1 \\ 0 \end{array} \right. \Rightarrow L_m = \left\{ \begin{array}{l} 0 \\ L_m^* \in (0,1) \\ 1 \end{array} \right. . \quad (19)$$

The uniqueness of L_m^* (which is obtained by $L_m^* = \arg\{L_m | \Lambda(L_m) = \kappa_L\}$) is, at least, not shown in the Autor and Dorn (2013) setting, and so multiple existences of L_m might happen under some specifications.

References

[1] Acemoglu, D., (2003) "Labor- and Capital-Augmenting Technical Change." *Journal of the European Economic Association*, Vol. 1, No. 1.

[2] Antras, P, (2004) "Is the U.S. Aggregate Production Function CobbDouglas? New Estimates of the Elasticity," B.E. Journal of Macroeconomics, Vol. 4, No. 1, pp. 1-36.

[3] Autor, D. H. and D. Dorn (2013) "The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market" *American Economic Review* 103(5): 1553-1597.

[4] Autor, D, Dorn, D, Katz, L, Patterson, C and Van Reenen J (2019) "The Fall of the Labor Share and the Rise of Superstar Firms, " Technical report, National Bureau of Economic Research, *(Quarterly Journal of Economics, Forthcoming)*,

[5] Bell, D., (1974) *The Coming of Post-Industrial Society.* New York: Harper Colophon Books.

[6] Blanchard, Oliver (1997) "The Medium Run," Brookings Papers on Economic Activity, Vol. 2, pp. 89-158.

[7] Chirinko, Robert S. (2008) "Sigma: The Long and Short of It." *Journal of Macroeconomics* 30 (2): 671-86.

[8] Chirinko, Robert S., Steven M. Fazzari, and Andrew P. Meyer. (2011) "A New Approach to Estimating Production Function Parameters: The Elusive Capital-Labor Substitution Elasticity." Journal of Business and Economic Statistics 29 (4): 587-94.

[9] Clark, C. (1941) *The Conditions of Economic Progress*, London, MacMillan & Co. Ltd.

[10] Cooley, T. F., and E. Prescott, (1995) "Economic Growth and Business Cysles" in T. F. Cooley ed. (1995) *Frontiers of Business Cycle Research*, Princeton University Press.

[11] Dao, Mai Chi, Mitali Das, Zsoka Koczan, and Weicheng Lian (2017) "Why Is Labor Receiving a Smaller Share of Global Income? Theory and Empirical Evidence, " Technical report, International Monetary Fund.

[12] Galbraith, J. K., (1958) *The Affluent Society.* Boston, Houghton Mifflin.

[13] Herrendorf, Berthold, Christopher Herrington, and Akos Valentinyi. (2013) "Sectoral Technology and Structural Transformation." Center for Economic Policy Research Discussion Paper 9386.

[14] Hobraba, N., and S. Kuwahara. (2020) "An Economic Development Model with Education and Industriousness." mimeo

[15] Jones, C. I. (2016) "The Facts of Economic Growth," in J. B. Taylor, H. Uhlig ed., *Handbook of Macroeconomics*, Volume 2A, October 2016.

[16] Karabarbounis, L. and B. Neiman (2014) "The Global Decline of the Labor Share," The Quarterly Journal of Economics, Vol. 129, No. 1, pp. 61?103, feb,

[17] Keynes, J. M., (1930) "Economic Possibilities for Our Grandchildren," in *Essays in Persuasion* (New York: Harcourt Brace, 1932), 358-373

[18] Keynes, J. M., (1939) "Relative Movements of Real Wages and Output," Economic Journal, Vol. 49, No. 193, pp. 34-51.

[19] Lawrence, Robert Z. (2015) "Recent Declines in Labor's Share in US Income : A Preliminary Neoclassical Account," National Bureau of Economic Research Working Paper 21296, DOI : `http://dx.doi.org/10.3386/w21296`.

[20] Maddison, A., (1982) *Phases of Capitalist Development.* Oxford: Oxford University Press.

[21] Mankiw, G. (2015) "Yes, $r > g$. So What?" *American Economic Review: Papers & Proceedings*, 105(5): 43-47

[22] Oberfield, Ezra, and Devesh Raval. (2014) "Micro Data and Macro Technology." National Bureau of Economic Research Working Paper 20452.

[23] Petty, W. (1690) *Political Arithmetick*, London: R. Cravel

[24] Piketty, Thomas. (2014) *Capital in the Twenty-first Century.* Cambridge, MA: Belknap Press of the Harvard University Press.

[25] Schwellnus, C., M. Pak, P. Pionnier and E. Crivellaro (2018) "Labour share developments over the past two decades: The role of technological progress, globalisation and "winner-takes-most" dynamics" OECD Economics Department Working Papers, No. 1503

[26] Summers, L., (2014) "Thomas Piketty is right about the past and wrong about the future" *The Atrantic*, 16 May.

[27] Toffler, Alvin, (1980) *The Third Wave.* Bantam Books.

[28] Uzawa, H. (1961) "Neutral Inventions and the Stability of Growth Equilibrium." *Review of Economic Studies* 28: 117-24.

[29] Vries, de J. (1994) "The Industrial Revolution and the Industrious Revolution," *The Journal of Economic History* Vol. 54, No. 2, Papers Presented at the Fifty-Third Annual Meeting of the Economic History Association (Jun., 1994), 249-270.

[30] World Bank (1993) *The East Asian miracle: Economic growth and public policy.* New York: Oxford University Press.