An Economic Growth Model with Education and Industriousness[1]

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Abstract

This study investigates the relationships between long-run growth, education, and "industriousness" using an extended Uzawa-Lucas model with labor-leisure choice, where "industriousness" is captured by the propensity for labor-leisure choice. The extension shifts from economic stagnation to long-run economic growth through the structural change of "industriousness," a growth path within the de Vries "industrious revolution." A domain that generates multiple steady states exists in the middle range of the industriousness parameter, implying the existence of middle income traps. Although the range is narrow, it can be broadened by, for example, higher population growth.

Keywords:

Industriousness; economic growth; Uzawa-Lucas model; labor-leisure choice; multiple steady states; middle-income trap

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1. Introduction

Although there is substantial consensus that education and educational productivity are key forces in economic growth (for example, Lucas 1988), education and economic growth are also associated with a theoretical puzzle. Some researchers have shown empirically that education does not always enhance growth in developing countries. Although several economists have shown that growth and schooling are highly correlated across countries, Bils and Klenow (2000) have found that the impact of schooling on growth explains less than one-third of the empirical cross-country relationship. Pritchett (2001) highlights the dwindling output of education, based on data from developing countries, by asking, "Where has all the education gone?" According to Benhabib and Spiegel (1994), there is no correlation between schooling duration and the per capita GDP growth rate.[4]

Another puzzle involves education and economic growth. Dore (1965, 1978) presents the historical fact that the literacy rate in Japan during the Edo period (1600–1868) compared favorably to that of Britain and France. Edo-period Japan had a high literacy rate but experienced delayed economic growth, whereas Britain and France experienced high economic growth. Thus, both contemporary and historical facts require us to revisit the mechanisms of human capital accumulation.

There appears to be a very important relationship between labor time and

[4] Recently, some studies, including Hanushek and Woessmann (2012), have reaffirmed the positive relationship between human capital and economic growth by using more elaborate and extensive data—beyond mere school-enrollment figures. These results imply that, although the endogenous-growth model with human capital accumulation remains effective, additional factors must also be considered.

economic growth, especially during the early stages of the process. However, economic growth theory traditionally focuses on poverty and unemployment (e.g., Lewis 1954; Fei and Ranis 1964; Jorgenson 1967). For this reason, few theoretical studies have found evidence supporting the relationship between labor supply and economic growth.[5]

One example can be found in Morse's (1917) reference to lazy Japanese workers. A similar caricature drawn by Charles Wirgman, entitled "Japanese at work," caricatures Meiji-era Japanese people by depicting them smoking in the middle of the workday (see http://kuwahara.soregashi.com/uls00.html#pic1). Of course, this is not a special case in Japan. In general, the shift toward industrial labor is a key factor, enabling the beginning of industrialization. In describing leisure in pre-modern societies, Sombart (1913) notes that a high proportion of craftspeople did not work more than they needed to achieve an ordinary standard of living, while Bavarian miners had many holidays.

In contrast, laborers in capitalist industrial societies after the Industrial Revolution had to work long hours. In 1831, Carlyle, a famous critic of the dismal science, described people being hurried during this period (Williams 1973). We also find descriptions of the long work hours of the laboring class after the Industrial Revolution in the works of Engelce (1844–1845), Sombart (1913), and others. In short, after the Industrial Revolution, workers began to work longer hours and had less leisure time. In a more modern study, Thompson (1968) reveals an important relationship between time and industrial capitalism. As Corbin (1995) notes, the Industrial Revolution triggered an adjustment in the allocation of time between labor and leisure.

[5]Some studies, which adopt the efficiency-wage hypothesis (e.g., Dasgupta 1993; Ray 1998) relate labor supply to growth. However, they relate labor quality and effort to economic growth. The present model focuses on quantitative labor time.

To analyze the puzzle between educated human resources and economic growth, we use a theoretical approach to analyze the changing relationship between time consciousness and economic growth. Specifically, this study uses the representative Uzawa-Lucas model (Uzawa 1965; Lucas 1988), [6] which links human capital accumulation to economic growth. Because the simple version of the Uzawa-Lucas model cannot account for this phenomenon, we add another factor to the model. This factor implies that there is another human resource supply mechanism in addition to the human capital accumulated through education. Therefore, we use the Uzawa-Lucas model with the labor-leisure choice developed by Benhabib and Perli (1994) and Ladrón-de-Guevara et al. (1997, 1999) to solve the puzzle and derive the interesting dynamic properties predicted by the extended model.

The historical description seems to imply that this period was not a one-time regime switch because the labor supply changed gradually, nor was it a converging process of success. Industrial Revolutions emerge at particular times during the growth process of every country. By using the term, "industrious revolution," suggested by de Vries (1994), the present study focuses on the gradual change of one deep parameter that determines labor-supply attitude, namely, "industriousness."

The current literature includes some interesting research. For example, Iacopetta (2010) developed a model in which innovation may commence before human capital accumulation. This reflects the fact that education began in England later, during the

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[6]Our method implicitly assumes that economic agents behave rationally through and before periods of economic growth. This assumption is supported by, for example, Chayanov (1966), who shows that peasants under czarism in Russia followed economic principles. The view of peasants presented in Schultz (1975) relates closely to the view of peasants in developing countries.

Industrial Revolution. Peretto (2015) built an economic growth model that includes a constantly growing labor supply, takeoff, and convergence. The model replicates the observed S-shaped growth rate. In contrast, this study presents a simple model without technological progress that contains an endogenous labor-leisure choice. It reveals the regime switch derived from the "industriousness" and the non-uniqueness of the economic growth path in middle-income economies. We consider shortened working hours during the period of economic growth – that is, *after* the Industrial Revolution – due to the change in the industriousness parameter. In short, according to Fouquet and Broadberry (2015), the European economy sustained long-run growth before the Industrial Revolution. It continued to confront the risk of decreasing growth, even after the Industrial Revolution. Thus, we derive the result that sustained growth cannot be attained unless people are sufficiently industrious, even with modern production technology.

Importantly, Ladrón-de-Guevara et al. (1997, 1999) conducted extensive inquiry into the model's properties. Their findings show that the model works well, whereas the parameter domains that yield multiple steady states are rather narrow, especially when closer to unity for the constant relative-risk aversion parameter. This study emphasizes the importance of attitudes toward labor supply or leisure preference on economic growth in the relationship between labor supply and education. Although Ladrón-de-Guevara et al. (1997, 1999) investigated the effects of various parameters, they did not adequately examine those of the shared parameter between consumption and leisure. This study revisits their model, focusing on the parameter that captures industriousness—the shared parameter between consumption and leisure. We show that the structural change in this shared parameter plays an important role in launching economic growth by highlighting the role it plays in generating steady state(s) and dynamic properties.

First, our model yields the basic result of the Uzawa-Lucas model, specifically that a combination of high educational efficiency and a low subjective discount rate is necessary for long-run positive growth. However, the combination of a high consumption-utility share and a low leisure-utility share also plays an important tertiary role in economic growth. Additionally, the typical parameter set seems to satisfy the conditions for the emergence of multiple steady states, and local and global indeterminacy.[7] Therefore, we revisit the prior result, that the model works well from the perspective of the wide industriousness parameter; during a period of expansion, the domain that yields indeterminacy also expands. We show theoretically that laborers able to work long hours make the "takeoff" (Rostow 1964) or "big spurt" (Gershenkron 1966) easier. In some cases, indeterminacy may occur during the process of economic growth.

Furthermore, the domain that yields multiple equilibria is wider when education has an externality. If economic growth is promoted through incremental industriousness, the economy always starts to develop across the domain that generates multiplicity, potentially causing a "middle-income trap" (Gill and Kharas 2007; Mitch 2005). Although shocks that hit developing economies are occasionally considered monetary shocks (if they involve money, as in a financial or currency crisis), this study's findings imply that two feasible equilibria and the local indeterminacy of dynamics, converging to a low-growth equilibrium, can cause a real phenomenon.

The remainder of this paper is organized as follows. The second section discusses

[7]Many studies relate endogenous labor supply to indeterminacy, including the pioneering study by Benhabib and Farmer (1994) and a relatively recent one by Farmer (2013).

historical labor time and growth data. The third section describes the model and derives equations describing the economy. The fourth section analyzes the dynamics and obtains the results for the steady-state properties, and the final section concludes the paper.

2. Historical data on labor and growth

In this section, we provide an overview of the historical facts and theoretical framework outlined in the previous section, confirming the discussion direction using historical data. First, we present data showing that human capital is not the most important factor during the earliest stage of economic growth. Second, we present data proving that economic growth can begin even when an economy has insufficient human capital.

First, we present evidence that sustained economic growth can occur even when an economy lacks sufficient human capital, namely, that British laborers during the Industrial Revolution had a relatively low literacy rate, while Japanese laborers in the pre-Industrial Revolution period were highly educated. The educational ability and growth rate in Japan are compared with the relationship between educational ability and growth rate in Britain.

Cipolla (1969) estimated the British illiteracy rate by analyzing whether brides and grooms were able to sign their own names at weddings. As shown in Table I, drawn from Cipolla (1969), the British illiteracy rate was 33% in 1841 (the literacy rate was 67%) after the Industrial Revolution. Therefore, we can conclude that Britain experienced the Industrial Revolution without fully accumulating human capital, as captured by the (il)literacy rate.

year	bloom	bride
1841	0.33	0.48
1842	0.32	0.48
1843	0.33	0.49
1844	0.32	0.49
1845	0.33	0.50
1846	0.33	0.48
1847	0.31	0.46
1848	0.31	0.45
1849	0.31	0.46
1850	0.31	0.46

Table I. Illiteracy rate of bride and groom in England and Wales (from Cibolla 1969).

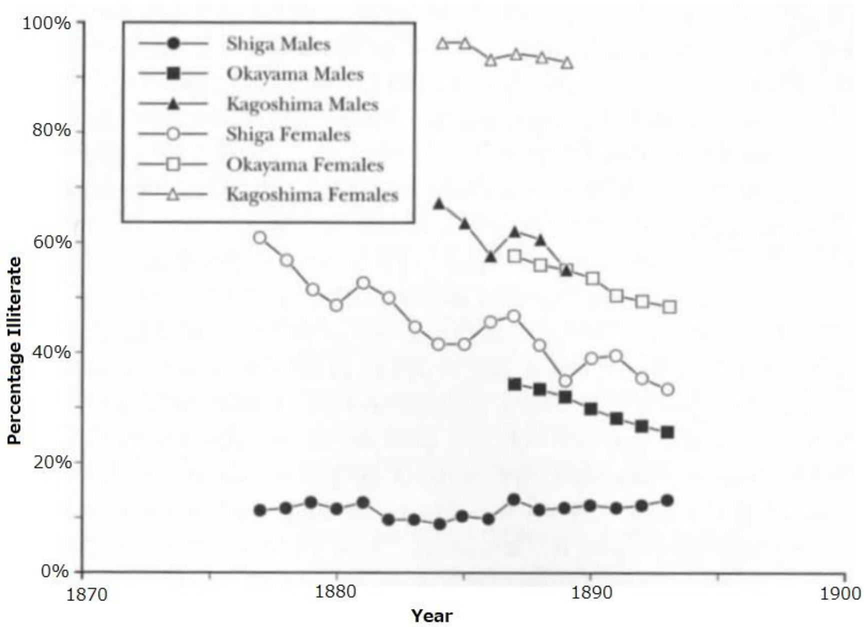


Figure 2. Japanese Male/female illiteracy in the Early Meiji Era Source: Monbusho nenpo (文部省年報)

However, education does not always lead to economic growth, as the data from Japan exemplifies. According to Dore (1965), “There is no doubt that the literacy rate in Japan in 1870 was considerably higher than in most underdeveloped countries today.”

Furthermore, as Fig. 2 shows, recent studies tend to indicate large area differences in Japan. For example, Shiga Prefecture, in the Kinki region, was said to be a relatively advanced province that entered the manufacturing stage during the Edo period. This restrictive investigation shows a rather low literacy rate in Kagoshima Prefecture, a non-advanced local area. Similar results have also been reported in Rubinger’s (2007) study, which rearranges the results of the Ministry of Education’s 1881 survey on Tokiwa Village, a non-advanced local area in Nagano Prefecture (see Table II). This analysis investigates the literacy rate in 1881 by grouping individuals into Edo-period decades by school age. We show that these individuals had already attained a high educational level during the Edo era.

Table II. Literacy rate in Tokiwa-Village (Rubinger 2007).

Age at 1881 a literacy test	70–79	50–59	30–39	10–19
Decade when examinees ware of school age (6–13)	1810s	1830s	1850s	1970s
1 Unable to read or write	51	48	35	24
2 Able to write name and address	25	33	39	48
3 Able to keep daily accounts	9	12	19	20
4 Able to read ordinary materials and fill out simple financial forms	9	1	5	3
5 Able to handle ordinary transactions	0	3	1	2
6 Able to read government documents and newspaper editorials	6	3	1	3

These phenomena have made it possible to estimate the accumulated extent of highly educated human resources in Japan as early as the Edo Period, even though Japan did not realize sustained economic growth during that era, in line with the argument proposed by Mitch (2005).

Second, this study reveals increasing labor time during the early period of economic growth. For example, Voth (1998) conducted an elaborate analysis of data from 19th century England (see Table III), demonstrating a tremendous increase in work hours during this period. Voth (1998) also notes that work hours began earlier and ended later. Combining Tables I and III shows that the most representative version of the Industrial Revolution (in Britain) was implemented by an increasingly hard-working, but not fully literate, labor force.

Table III. Working hours per year in England (Voth 1998).

year	1760	1800	Difference
Lower boud	2,288	3,366	+1,078
Upper bound	2,631	3,538	+907

It is difficult to confirm the above argument precisely because the data comes from a period before economic and social data were ordered systematically. However, we can grasp the level of support from these fragmented data, corroborating our argument.

In Japan, the increase in labor time during the early economic growth process can be confirmed by data from the construction of the Nagasaki Dock (see Table IV, cited from Saito 1998). Although the period in question is 1900–1940, which was not the earliest period of economic growth but followed the initial takeoff, we can nevertheless confirm the impact of increasing labor time on the economic growth process.

Table IV. Transition of working time in Nagasaki building dock in Japan (from Saito 1998).

periods	labor days per a month	labor hours per a day	labor hours per a month
1900	21.29	10.05	214.0
1905	25.50	10.08	257.0
1910	25.17	9.39	236.3
1915	24.80	10.62	263.4
1920	24.77	9.11	225.7
1925	24.42	9.74	237.9
1930	24.47	9.99	244.5
1935	24.75	11.17	276.5
1940	23.60	11.01	259.8

At a minimum, we can conclude that increasing labor supply played an important role during the early stages of economic growth. Such phenomena have been indirectly reported by Abramovitz and David (1973) and Hayami and Ogasawara (1999), who analyzed the regime switch from factor accumulation to total factor productivity (TFP) growth using data from the U.S. and Japan, respectively.

3. The Model

As stated above, this study uses a simple version of the model developed by Ladrón-de-Guevara et al. (1997, 1999). By defining c and l as consumption and leisure respectively, the objective function of a representative household is specified as $U = \int_0^\infty u\,(c, l; \phi) e^{-\rho t} dt$, where $u(c, l, \phi)$ is the instantaneous utility function, and $\rho (> 0)$ and ϕ denote the subjective discount rate and a parameter between c and l, respectively.

As noted in the Introduction, we have evidence that per capita leisure time generally remained constant during the post-war period. We also know that real wages increased steadily during the post-war period. Taken together, these two observations imply that the elasticity of substitution between consumption and leisure should be close to unity (Cooley and Prescott 1995). In other words, we can use a Cobb-Douglas-type composite input on the instantaneous utility function, which is defined as $c^{\phi}l^{1-\phi}$. An instantaneous utility function is then assumed to have constant relative risk aversion (CRRA), beginning with the following:

$$u(c,l;\phi) = \frac{(c^{\phi}l^{1-\phi})^{1-\sigma} - 1}{1-\sigma}. \tag{1}$$

Later, we assume a log linear for a simple dynamical analysis. In the model, the parameter that determines labor supply propensity is a utility shared parameter between consumption (defined by c) and leisure (defined by l). We define consumption and leisure as ϕ and $1-\phi$, respectively, showing that the increment of labor supply is a descent from the leisure parameter, namely, the increment of the consumption parameter ϕ. Hence, we view ϕ as the parameter of "industriousness." The present study considers the industriousness parameter ϕ, which exogenously determines the utility share between consumption and leisure, even when given historically or habitually, and attitudes toward labor time (undergoing gradual change), analyzing their dynamic effects.

We assume the production structure with a Cobb-Douglas production function, with human and physical capital as inputs and without externalities. We also follow the simple, typical Uzawa-Lucas structure, in which final goods are invested as physical capital or consumed as consumption goods, and human capital is accumulated by human capital investment in the education sector with constant returns. Thus, the resource constraints are given as follows:

$$\dot{k} = \underbrace{k^{\beta}(uh)^{1-\beta}}_{:=y} - c - (n + \delta_k)k, \quad \beta \in (0,1), \delta_k > 0 \tag{2}$$

$$\dot{h} = b(1 - u - l)h - \delta_h h, \quad b > 0, \quad b > \delta_h, \tag{3}$$

where y, k, h, u, and δ_k, denote per capita output, per capita physical capital, per capita human capital, the allocation share of human capital for production, and the physical capital-depreciation rate, respectively. Note that we can identify the aggregate and per capita values because we assume a constant population. For δ_h, it is usually treated as the human capital depreciation rate, with the restriction $\delta_h > 0$. However, we can interpret this broadly, as capturing the positive spillover effects on education. In this case, $-\delta_h$ becomes positive, whereas δ_h becomes negative. Thus, we do not restrict our analysis to a positive δ_h here.

Combining perfect competition and Equation (2), the interest rate and wage are given as $r = \beta y/k$ and $w = (1 - \beta)y/(uh)$, respectively. We then introduce labor-leisure choice, denoting the time devoted to leisure as l. In this study, human capital is used for final goods production, human capital accumulation, and leisure. These time inputs are represented as u, $1 - u - l$, and l, respectively, where $u, l,$ and $1 - u - l \in [0,1]$ must hold.

The household is assumed to maximize the utility integration U, subject to the budget constraint $\dot{k} = rk + wuh - c - (n + \delta_k)k$. Thus, the Hamiltonian function in this study is as follows:

$$\mathcal{H} = u(c, l) + \lambda_k\{rk + wuh - c - (n + \delta_k)k\} + \lambda_h\{b(1 - u - l)h - \delta_h h\},$$

where λ_k and λ_h denote the shadow prices of physical and human capital, respectively. The Hamiltonian yields the following optimal conditions:

$$\phi c^{\phi(1-\sigma)-1} l^{(1-\phi)(1-\sigma)} = \lambda_k, \tag{4}$$

$$(1-\phi) c^{\phi(1-\sigma)} l^{(1-\phi)(1-\sigma)-1} = \lambda_h b h, \tag{5}$$

$$\lambda_k w = \lambda_h b, \tag{6}$$

$$\rho\lambda_k - \dot{\lambda}_k = \lambda_k (r - n - \delta_k), \tag{7}$$

$$\rho\lambda_h - \dot{\lambda}_h = \lambda_k w u + \lambda_h \{b(1-u-l) - \delta_h\}. \tag{8}$$

The transversality conditions are as follows:

$$\lim_{t\to\infty} e^{-\rho t} \lambda_{kt} k_t = 0, \quad \text{and} \quad \lim_{t\to\infty} e^{-\rho t} \lambda_{ht} h_t = 0.$$

From $w = (1-\beta)y/(uh)$ and Equations (2) and (6), we have the following:

$$\frac{\dot{\lambda}_h}{\lambda_h} - \frac{\dot{\lambda}_k}{\lambda_k} = \beta\frac{\dot{k}}{k} - \beta\frac{\dot{u}}{u} - \beta\frac{\dot{h}}{h}. \tag{9}$$

From Equations (4) and (7), we obtain the following:

$$\rho - \frac{\dot{\lambda}_k}{\lambda_k} = \rho + \frac{\dot{c}}{c} - \xi\left(\frac{\dot{c}}{c}, \frac{\dot{l}}{l}; \phi, \sigma\right) = r - \delta_k - n, \tag{10}$$

where $\xi(\dot{c}/c, \dot{l}/l; \phi, \sigma) \equiv \phi(1-\sigma)(\dot{c}/c) + (1-\phi)(1-\sigma)(\dot{l}/l)$. It should be noted that $\xi(\dot{c}/c, \dot{l}/l; \phi, 1) = 0$ and $\xi(0, 0; \phi, \sigma r) = 0$.

From Equations (5), (6), and (8), we have the following:

$$\rho - \frac{\dot{\lambda}_h}{\lambda_h} = \rho + \frac{\dot{h}}{h} + \frac{\dot{l}}{l} - \xi = b(1-l) - \delta_h. \tag{11}$$

Substituting Equation (3) into (11) yields:

$$\frac{\dot{l}}{l} = bu - \rho + \xi. \tag{12}$$

Using Equation (2) and $r = \beta k^{\beta-1}(uh)^{1-\beta}$, and defining $q := c/k$, we obtain the following:

$$\frac{\dot{k}}{k} = \frac{r}{\beta} - q - \delta_k - n. \tag{13}$$

Substituting Equations (3), (10), (11), and (13) into Equation (9), we obtain the following:

$$\frac{\dot{u}}{u} = \frac{1-\beta}{\beta}\{b(1-l) - \delta_h + \delta_k + n\} + bu - q. \tag{14}$$

Therefore, Equations (3), (13), and (14) and the definition of r yield the following dynamic equation:

$$\frac{\dot{r}}{r} = \frac{1-\beta}{\beta}\{b(1-l) - \delta_h - (r - \delta_k - n)\}. \tag{15}$$

On q, combining Equations (4), (5), and (6) gives q as a function of l, u, and r, as follows:

$$q = \frac{\tilde{\beta}}{\tilde{\phi}}\frac{r\,l}{u}(\equiv q(l,r,u)). \tag{16}$$

where $\tilde{\phi} \equiv \frac{1-\phi}{\phi}$ and $\tilde{\beta} \equiv \frac{1-\beta}{\beta}$; $\tilde{\phi}$ have a high value when the share of leisure in the utility function is high. Additionally, $\tilde{\beta}$ indicates the efficient ratio of human and physical capital.

Thus, the dynamic Equations (12), (14), and (15) describe a system that consists of $\{l(t), u(t), r(t)\}$, where the two variables u and l are jumpable and control variables. Here, r functions as a state variable that is changed by controlling u and l through the dynamics of k and h.

4. Dynamics and Stability

For simplicity, we assume that $n = \delta_k = \delta_h = 0$ and $\sigma = 1$. The latter simplification $\theta = 1$ makes the utility function a log-linear function, while $\Omega = 0$ and the latter result of $\Omega = 0$ simplify the dynamic properties. Then, utility function (1) is converted into the following:

$$u(c,l) = \phi \log\ c + (1-\phi) \log\ l, \qquad \phi \in (0,1).$$

1. Steady States

Next, we examine dynamics and stability. First, we obtain the steady states of the system. Imposing $\dot{l} = \dot{u} = \dot{r} = 0$ on Equations (12), (13), (14), (15), and (16), we obtain the following steady-state values:

$$u^* = \frac{\rho}{b}, \quad r^* = b(1-l^*) + \nu(\equiv g(l)), \quad \text{and} \quad q^* = \tilde{\beta} r^* + \rho, \tag{17}$$

where $\nu \equiv n + \delta_k - \delta_h$. Note that the steady state shown in Equation (17) is related to that of positive education $(1 - u^* - l^* > 0)$. If even one condition fails to occur, the system cannot take the path to long-run positive growth and is stuck in a no-growth trap, characterized by $r^* = \rho$ and $u^* + l^* = 1$. In this case, the model is reduced to a simple Ramsey model with leisure-labor choice.

From $u^* = \frac{\rho}{b}$ above, $u^* \in (0,1)$ and Equation (12), we have the usual condition for non-negativity:

$$b > \rho. \tag{18}$$

By changing q^* from Equation (17) into $q(r,u,l)$ from Equation (16), we obtain the necessary relationship between r^* and l^*:

$$r^* = \frac{\tilde{\phi}\rho^2}{\tilde{\beta}(b\,l^* - \tilde{\phi}\rho)}\,(\equiv v(l)). \tag{19}$$

By uniting $g(l)$ and $v(l)$, we can obtain Fig. 3.

In (19), $r > 0$ implies $bl^* - \tilde{\phi}\rho > 0$ from Equation (19) and $b(1-l) + \nu > 0$ from $r = g(l)$, which yield $l^* > \frac{\tilde{\phi}\rho}{b}$ and $l^* < 1 - \frac{\nu}{b}$, respectively. From Equation (17), $u^* = \frac{\rho}{b}$ implies $l^* \leq \underline{a}rl\left(\equiv 1 - u^* = 1 - \frac{\rho}{b}\right)$. Thus, it is necessary for the feasible condition on l^* to hold that $\frac{\tilde{\phi}\rho}{b} < l^* < \min\left[1 - \frac{\rho}{b}, 1 + \frac{\nu}{b}\right]$. This condition implies that $\frac{\tilde{\phi}\rho}{b} < \min\left[1 - \frac{\rho}{b}, 1 + \frac{\nu}{b}\right]$ is necessary, which can be rewritten as follows:

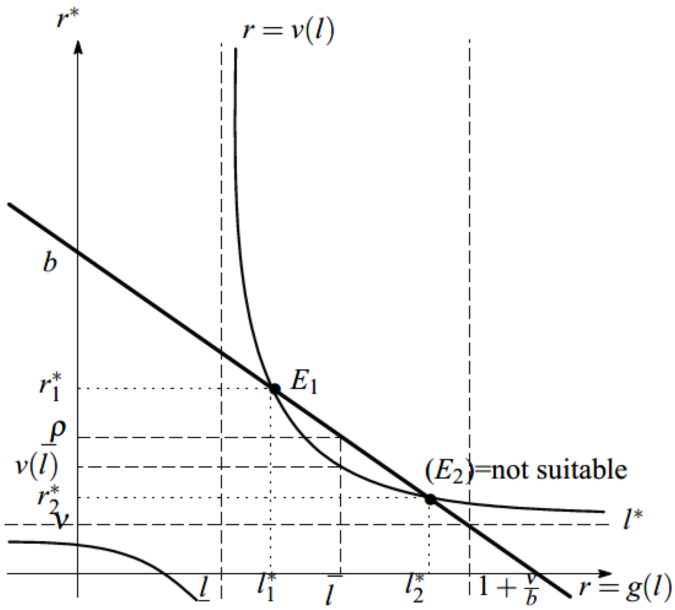
$$b > \max\left[(1+\tilde{\phi})\rho,\, \tilde{\phi}\rho - \nu\right]. \tag{20}$$

As shown in Fig. 3, the intersections of $g(l)$ and $v(l)$ provide the values of r^* and l^* in the steady state. The r^* value given in Equation (19) must be positive as a necessary condition for a steady state; therefore, $l > \underline{l}$ must hold, where $\underline{l} \equiv \frac{\tilde{\phi}\,\rho}{b}$ and $l = \underline{l}$ is an asymptote of the function $v(l)$ for the smaller l, and $r = \nu$ is the asymptote for the larger l. Although Equation (17) implies that the necessary nonnegative condition for r^* is $r^* < b$, Fig. 3 shows that this condition is always satisfied. From this discussion, we obtain the following:

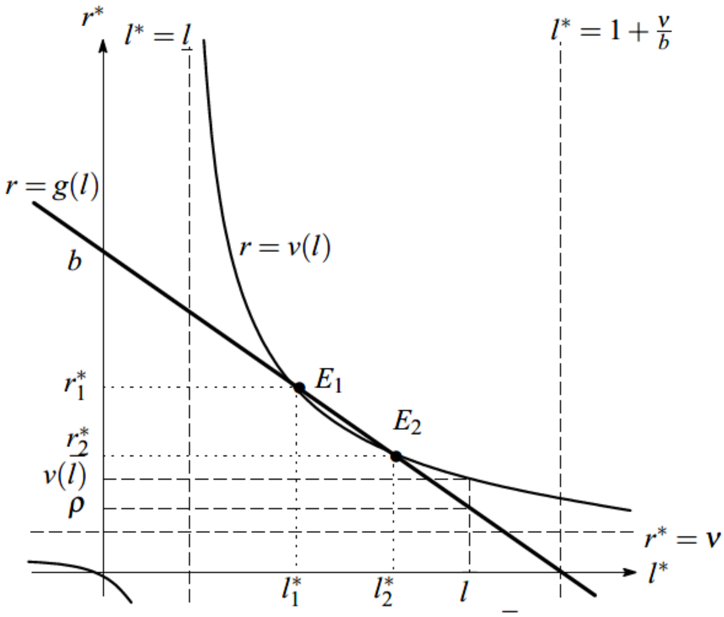
$$\begin{aligned} b &> \max\left[\,(1+\tilde{\phi})\rho,\ \tilde{\phi}\rho - \nu,\ \rho\right] \\ &(= \max\left[\,(1+\tilde{\phi})\rho, \tilde{\phi}\rho - \nu\right]) \end{aligned} \tag{21}$$

To obtain the equilibrium r^*, we eliminate l using $r = g(l) = b(1-l) + \nu$ and $r = v(l)$, obtaining the following:

$$\Omega(r^*) \equiv r^{*2} + r^*(\tilde{\phi}\rho - b - \nu) + \frac{\tilde{\phi}}{\tilde{\beta}}\rho^2 = 0. \tag{22}$$



(a) The case of $\underline{l} < l_1^* < \bar{l} < l_2^*$



(b) The case of $\underline{l} < l_1^* < l_2^* < \bar{l}$

Figure 3. The equilibrium allocation of l^* and r^*

Thus, we transform the existence of the model's solution(s) into the existence of a positive (and less-than-b) root(s) of the quadratic Equation (22). Fig. 4 depicts the graph for

Equation (22). Using the rule of solutions, we can obtain the analytical r, as follows:

$$r_1^* = \frac{b + \nu - \tilde{\phi}\rho + \sqrt{D_\Omega}}{2}, \quad \text{and} \quad r_2^* = \frac{b + \nu - \tilde{\phi}\rho - \sqrt{D_\Omega}}{2}, \tag{23}$$

where D_Ω denotes the discriminant of the quadratic Equation (22) and is given as follows:

$$D_\Omega \equiv (\tilde{\phi}\rho - b - \nu)^2 - 4\frac{\tilde{\phi}\rho^2}{\tilde{\beta}}. \tag{24}$$

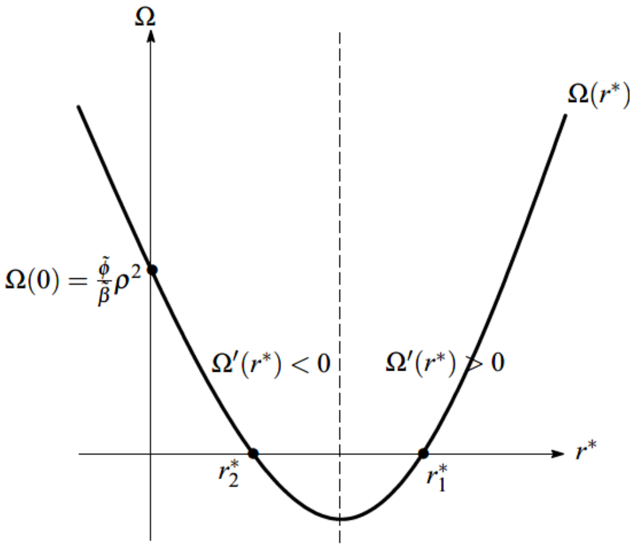
Since Equation (22) is a quadratic equation, if it has a real root, then the two roots lie on both sides of the axis of the quadratic equation $\Omega(r)$, where $\Omega'(\cdot) = 0$ gives the value of r on the axis. Because Equation (22) has a positive coefficient in the second-order term, the root on the left side of the axis is related by $\Omega'(r^*) < 0$ and vice versa. Thus, we have two different real-number solutions in this situation, $r_1, r_2, (r_1 > r_2)$, both of which satisfy the following property:

$$\Omega'(r) = 2r^* + \tilde{\phi}\rho - b - \nu \begin{Bmatrix} > \\ < \end{Bmatrix} 0 \quad \text{for} \quad r = \begin{cases} r_1 \\ r_2 \end{cases}. \tag{25}$$

Thus, under the condition of the existence of real roots, the graph of $\Omega(r)$ appears as shown in Fig. 4.

Next, we investigate the conditions of the existing real roots. Since $\Omega(0) = \frac{\tilde{\phi}}{\tilde{\beta}}\rho^2 > 0$, i) $\Omega'(r^* = 0) < 0$, and ii) $D_\Omega > 0$ are necessary for the existence of the positive roots in Equation (22), the condition i) yields a positive root(s) if it/they exist(s), and condition ii) verifies the existence of positive root(s) if it/they exist(s).

Figure 4. The form of $\Omega(r^*)$



Condition i) yields $\Omega'(r^* = 0) = \tilde{\phi}\rho - b < 0$, namely $b > \tilde{\phi}\rho$, which is consistent with the positive condition for r^* derived from Equation (19). If $b < \tilde{\phi}\rho$, the equilibrium related to positive long-run growth does not exist. In the case of $b > \tilde{\phi}\rho$, condition ii) yields the following:

$$b > \tilde{\phi}\rho\left(1 + \frac{2}{\sqrt{\tilde{\phi}\,\tilde{\beta}}}\right) - \nu. \tag{26}$$

The case of $b < \tilde{\phi}\rho$ immediately contradicts at least Equation (20). The equilibrium related to positive long-run growth therefore cannot be feasible in this case. Thus, Equations (21) and (26) are the necessary conditions that the economy should satisfy in a steady state with a positive growth rate.

Lemma 1

From the existence of a steady state with positive education and positive long-run growth, *we derive the following:*

$$b > \max\left[(1+\tilde{\phi})\rho,\ \tilde{\phi}\rho+\nu,\ \left(1+\frac{2}{\sqrt{\tilde{\phi}\,\tilde{\beta}}}\right)\tilde{\phi}\rho-\nu\right];$$

otherwise, the economy is stuck in the steady state related to no education and a 0 *long-run growth rate, characterized by* $r^* = \rho$.

We note that economies in a steady state are characterized by $r^* = \rho$, the state without human capital accumulation, which is basically the same as the simple Ramsey model with labor and leisure choices and constant human resources.

The former two conditions in Lemma 1, $b > (1+\tilde{\phi})\rho$ and $b > \tilde{\phi}\rho + \nu$, immediately become the following:

$$\phi > \max\left[\frac{\rho}{b}, \frac{\rho}{b+\nu+\rho}\right].$$

The last condition in Lemma 1, $b > \tilde{\phi}\left[1+\frac{2}{\sqrt{\tilde{\beta}\tilde{\phi}}}\right]\rho+\nu$, yields the following quadratic inequality for $\sqrt{\tilde{\phi}}$:

$$\tilde{\phi} + \frac{2}{\sqrt{\tilde{\beta}}}\sqrt{\tilde{\phi}} - \frac{b+\nu}{\rho} < 0.$$

This quadratic inequality and non-negativity of $\sqrt{\tilde{\phi}}$ produce the following solution:

$$0 < \sqrt{\tilde{\phi}} < \frac{-1+\sqrt{1+\tilde{\beta}\left(\frac{b+\nu}{\rho}\right)}}{\sqrt{\tilde{\beta}}}.$$

Since $-1+\sqrt{1+\tilde{\beta}\{(b+\nu)/\rho\}} > 0$, the inequality has a real solution interval. We rewrite this condition as follows:

$$(1>)\phi > \frac{1}{1+\frac{1}{\tilde{\beta}}\left(2+\tilde{\beta}\frac{b+\nu}{\rho}-2\sqrt{1+\tilde{\beta}\frac{b+\nu}{\rho}}\right)} (:= \underline{\phi}),$$

where we can easily show $\underline{\phi} > 0$ using $2 + X > 2\sqrt{1+X}$ for $X > 0$. We should note that $\underline{\phi} \in (0,1)$, $\frac{\partial \underline{\phi}}{\partial b} < 0$, and $\frac{\partial \underline{\phi}}{\partial \rho} > 0$. These derivatives imply that the lower limit of ϕ is smaller under the higher educational efficiency b and the lower subjective discount rate ρ.

From the above discussion, we can derive the following corollary to Lemma 1:

Corollary

From the existence of a steady state with positive education and positive long-run growth, we derive the following:

$$(1>)\phi > \max\left[\frac{\rho}{b}, \frac{\rho}{b+\nu+\rho}, \underline{\phi}\right]. \tag{27}$$

Thus, we obtain the condition of a real root of $\Omega(r) = 0$. Here, we provide a numerical discussion. We impose $\beta = 0.33..$ and $\nu = 0$, where β is a representative value and $\nu = 0$ yields $\max\left[\rho/b, \rho/(b+\nu+\rho)\right] = \rho/b$, which makes the analysis easier. Table V provides the numerical results, where the references for b are as follows: Ladrón-de-Guevara et al. (1997) used $b = 0.769$, Ladrón-de-Guevara et al. (1999) used $b = 0.25$, and Mino (2003) used $b = 0.15$. Then, we present the result below:

Result 1 *Under a plausible specification of* $\boldsymbol{b}$, $\boldsymbol{\rho}$, *and* $\boldsymbol{\beta}$, *the equation* $\boldsymbol{\Omega}(\boldsymbol{r}^*) = \boldsymbol{0}$, *which yields the equilibrium interest rate(s), has a (comparatively) broad feasible domain on*

$\boldsymbol{\phi}$, and a broader domain trend for a lower $\boldsymbol{b}$ and lower $\boldsymbol{\rho}$.

b	$\rho = 0.05$	$\rho = 0.03$
0.769	$\underline{0.085}$ (0.065)	$\underline{0.049}$ (0.039)
0.25	$\underline{0.271}$ (0.200)	$\underline{0.163}$ (0.120)
0.15	$\underline{0.425}$ (0.333)	$\underline{0.271}$ (0.200)

Table V. Value of $\underline{\boldsymbol{\phi}}$ (and $\boldsymbol{\rho/b}$) and effective constraint.

Notably, in the preventative case where $\{\beta, b, \rho, \nu\} = \{0.33, 0.25, 0.05, 0.00\}$, we obtain $\underline{\phi} = 0.2714..$, and the condition in Equation (27) in the Corollary becomes $\phi \in (0.27149, 1)$; thus, we find that in a broad range of i, the Uzawa-Lucas model with labor-leisure choice has feasible steady states.

Next, we check the possibility of multiple steady states by investigating whether $r_i (i = 1,2)$, obtained in Equation (25) satisfies the feasibility conditions. From $bu^* = \rho$ in Equations (17) and (3) and the non-negativity of human capital accumulation, we have $g_H^* = b(1 - l^*) + \nu - \rho > 0$ and $l^* < 1 - u^* (= \bar{l})$. We also have the condition $r^* = b(1 - l^*) - \nu$ from Equation (15). Combining these two conditions, we again obtain the usual positive conditions as $r^* > \rho$ and $l^* < \bar{l}$.

Referring to Figs. 3 and 4, we have the following condition:[8]

$$\nu(\bar{l}) < \rho \quad \cdots \quad r^* = r_1$$

[8]Note that $\Omega(\rho) > (<) 0$ corresponds to $\nu(\bar{l}) > (<) \rho$.

$$v(\bar{l}) > \rho \quad \text{and} \quad \Omega'(\rho)\begin{Bmatrix}<\\>\end{Bmatrix}0 \quad \cdots \quad \begin{cases} r^* = r_1,\ r_2, \\ r^* = \rho. \end{cases}$$

These conditions $v(\bar{l})\begin{Bmatrix}<\\>\end{Bmatrix}\rho$ and $\Omega'(\rho)\begin{Bmatrix}<\\>\end{Bmatrix}0$ become, respectively, the following:

$$v(\bar{l})\begin{Bmatrix}<\\>\end{Bmatrix}\rho \iff b\begin{Bmatrix}>\\<\end{Bmatrix}\rho\left(1+\frac{\tilde{\phi}}{1-\beta}\right)(>\rho(1+\tilde{\phi})) \tag{28}$$

$$\iff \phi\begin{Bmatrix}>\\<\end{Bmatrix}\left[1+\left(\frac{b}{\rho}-1\right)(1-\beta)\right]^{-1} (\equiv \bar{\phi}),$$

and

$$\Omega'(\rho)\begin{Bmatrix}<\\>\end{Bmatrix}0 \iff b\begin{Bmatrix}>\\<\end{Bmatrix}\rho(2+\tilde{\phi})-\nu \tag{29}$$

$$\iff \phi\begin{Bmatrix}>\\<\end{Bmatrix}\frac{\rho}{b+\nu-\rho}.$$

Thus, we obtain the following result:[9]

Result 2 *We have the following pattern for the steady state(s):*

$$r^* = r_1 \quad \text{for} \quad b > \rho\left[1+\frac{\tilde{\beta}}{1-\beta}\right],$$

$$\text{which yields:} \quad 1 > \phi > \max\left[\frac{\rho}{b}, \frac{\rho}{b+\nu-\rho}, \bar{\phi}, \underline{\phi}\right] \tag{30}$$

$$r^* = r_1, r_2 \quad \text{for} \quad \rho\left[1+\frac{\tilde{\beta}}{1-\beta}\right] > b > \rho(2+\tilde{\phi})-\nu,$$

$$\text{which yields:} \quad \bar{\phi} > \phi > \max\left[\frac{\rho}{b}, \frac{\rho}{b+\nu-\rho}, \underline{\phi}\right]. \tag{31}$$

$$r^* = \rho \quad \text{for} \quad b < \rho(2+\tilde{\phi})-\nu,$$

$$\text{which yields:} \quad \max\left[\frac{\rho}{b}, \frac{\rho}{b+\nu-\rho}, \underline{\phi}\right] > \phi > 0. \tag{32}$$

---

[9]We note that the condition max $\left[\frac{\rho}{b+\nu+\rho}, \frac{\rho}{b+\nu-\rho}, \ldots\right]$ reduces to max $\left[\frac{\rho}{b+\nu-\rho}, \ldots\right]$

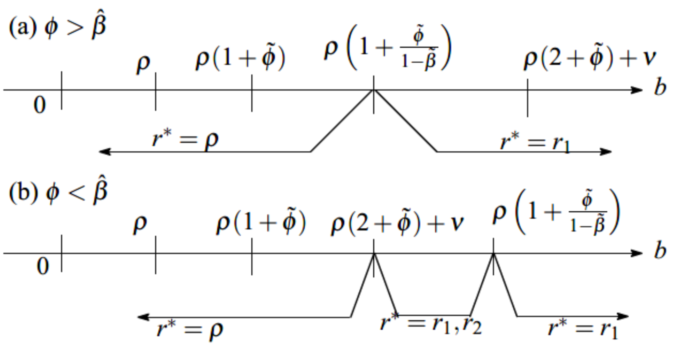
We note that Equation (31) is possible under the restriction $\rho\left(1+\frac{\tilde{\phi}}{1-\beta}\right) > \rho(2+\tilde{\phi}) + \nu$*, which is as follows:*

$$\phi > \beta\left(1 - \frac{\tilde{\beta}\nu}{\rho}\right)^{-1} \ (\equiv \hat{\beta}). \quad (33)$$

This is the condition for multiple r *values to exist. Furthermore, this* $\phi > \hat{\beta}$ *changes Equation (31) to* $b < (2+\tilde{\phi}) + \nu$*, while* $\phi > \hat{\beta}$ *changes Equation (31) to* $b < \rho\left(1 + \frac{\tilde{\phi}}{1-\beta}\right)$.

Under the assumption that $\nu = 0$, $\hat{\beta} = \beta$ holds; therefore, for the small ν relative to ρ, the threshold is around the capital share β. Fig. 5 illustrates the pattern of emergence of the steady state corresponding to the size of b.

Figure 5. The pattern of emergence of the steady states



The numerical results for $\{\beta, b, \rho. \nu\} = \{0.33..,0.25,0.05,0.00\}$ yield $\bar{\phi} = 0.272...$, $\underline{\phi} = 0.2714..$, and so on. Thus, we obtain the following conditions for ϕ:

$$r^* = r_1 \quad \text{for} \quad 1 > \phi > 0.2727.., \quad (34)$$

$$r^* = r_1, r_2 \ \ \text{for} \quad 0.2727.. > \phi > 0.2714.., \quad (35)$$

$$r^* = \rho \quad \text{for} \quad 0.2714.. > \phi > 0. \tag{36}$$

The domain with positive growth is $\phi > 0.2714$.... For a smaller ϕ; that is, a higher utility weight of leisure, the economy has only a steady state with no growth ($r^* = \rho$). Importantly, in the domain of ϕ, there is a very narrow non-empty set that is consistent with multiple steady states, as shown by Ladrón-de-Guevara et al. (1997, 1999). There is also a very narrow non-empty set that is consistent with the multiple steady states associated with capital share, represented by β in this study.

The following section shows the emerging pattern of steady states and investigates dynamic properties.

2. Dynamics and Stability

Next, we examine the dynamic system in this study. From Equations (12), (14), and (15), we derive the system using the following three dynamic equations:

$$\dot{l}(t) = (bu(t) - \rho)l(t) \tag{37}$$

$$\dot{u}(t) = \left[\tilde{\beta}\{b(1 - l(t)) + \nu\} + bu(t) - q(l(t), u(t), r(t))\right]u(t), \tag{38}$$

$$\dot{r}(t) = \tilde{\beta}\{b(1 - l(t)) + \nu - r(t)\}r(t). \tag{39}$$

From Equations (37)–(39), we derive the following linearized system:

$$\begin{pmatrix} \dot{l} \\ \dot{u} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} 0 & bl^* & 0 \\ -\tilde{\beta}\rho - \dfrac{\tilde{\beta}}{\tilde{\phi}}r^* & \rho + q^* & -\dfrac{\tilde{\beta}}{\tilde{\phi}}l^* \\ -\tilde{\beta}br^* & 0 & -\tilde{\beta}r^* \end{pmatrix} \begin{pmatrix} l - l^* \\ u - u^* \\ r - r^* \end{pmatrix},$$

where the relationship $\frac{q^*}{l^*}u^* = \tilde{\phi}\tilde{\beta}r^*$ is used in the derivation process. We represent the eigenvalues in this system as λ as the solution to the following characteristic equation:

$$\Psi(\lambda) = -\lambda^3 + Tr^*\lambda^2 - B^*\lambda + Det^*,$$

where Det^*, Tr^*, and B^* are defined as follows:

$$Det^* := -\frac{b\,\tilde{\beta}^2 l^* r^*}{\tilde{\phi}}\left(2r^* + \tilde{\phi}\rho - b\right)\begin{Bmatrix} < \\ > \end{Bmatrix} 0, \quad \text{for} \quad r^* = \begin{cases} r_1^* \\ r_2^* \end{cases},$$

$$Tr^* := \rho + q^* - \tilde{\beta}r^* = 2\rho > 0,$$

$$B^* := \begin{vmatrix} 0 & bl^* \\ -\tilde{\beta}\left(\rho + \frac{r^*}{\tilde{\phi}}\right) & \rho + q^* \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ -\tilde{\beta}br^* & -\tilde{\beta}r^* \end{vmatrix} + \begin{vmatrix} \rho + q^* & -\frac{\tilde{\beta}}{\tilde{\phi}}l^* \\ 0 & -\tilde{\beta}r^* \end{vmatrix}$$

$$= \tilde{\beta}\left[bl^*\left(\rho + \frac{r^*}{\tilde{\phi}}\right) - (\rho + q^*)r^*\right] = \tilde{\beta}\left[\rho\left(b + \frac{\rho}{\tilde{\beta}} + \tilde{\phi} - 2\right) - r^*(b - \rho)\right],$$

where we obtain r^* in Equation (23) and the sign of Det^* from Equation (25), and derive the last line of B^* using $\rho\left(r^* + \frac{\rho}{\tilde{\beta}}\right) = \frac{bl^*}{\tilde{\phi}}r^*$, which is itself obtained from $r = v(l)$.

When $r^* = r_1^*$, we immediately obtain the saddle-stable property from $Det^* < 0$ and $Tr^* > 0$. For $r^* = r_2^*$, the scheme of things is slightly more complicated. The combination of $Det^* > 0$ and $Tr^* > 0$ yields two possible combinations of eigenvalues: $\{+ + +\}$ and $\{+ - -\}$. To determine which case will emerge, we use the Routh-Hurwitz theorem, as follows:

Routh-Hurwitz Theorem. *The number of roots of the characteristic equation* $\boldsymbol{\varphi}(\boldsymbol{\lambda}) = \mathbf{0}$ *with positive parts is equal to the number of variations of sign in the following scheme:*

$$-1, \quad Tr^*, \quad -B^* + \frac{Det^*}{Tr^*}, \quad Det^*.$$

Proof: See, for example, Gantmakher (1960).

This theorem shows that when $-B^* + \frac{Det^*}{Tr^*} > 0$, then, the signs in the scheme are

–,+,+,+, and there is one sign change: as the number of positive roots is 1, the set of eigenvalues in the study model is given as $\{+,-,-\}$. When $-B^* + \frac{Det^*}{Tr^*} < 0$, the signs in the scheme are –,+,–,+, and there are 3 positive roots, showing that the system has a combination of eigenvalues $\{+,+,+\}$. As noted at the end of Section 2, the model has two control variables, u and l, and one state variable, r, so the combination of eigenvalues $\{+,-,-\}$ implies local indeterminacy, whereas that of $\{+,+,+\}$ is unstable.

Since $\frac{Det^*}{Tr^*} > 0$ holds, if $B^* < 0$ is satisfied then the value of $-B^* + \frac{Det^*}{Tr^*}$ is always positive. Hence, it is one of the sufficient conditions for our model to show local indeterminacy. Therefore, we seek the condition that yields $B^* < 0$, and then obtain $r^*(b-\rho) > \rho\left[b + \left(\frac{1}{\beta} + \tilde{\phi}\right)\rho - 2\right]$. Here, we again use the relationship $b > \rho$, which can be derived from the numerical examples $b = 0.25$ and $\rho = 0.05$, used by Ladrón-de-Guevara et al. (1999).[10] Solving $\tilde{\beta}\, r^*(b-\rho) > \rho\left[b + \left(\frac{1}{\beta} + \tilde{\phi}\right)\rho - 2\right]$ for r, we obtain the following inequality:

$$r^* > \frac{\rho}{b-\rho}\left[b + \left(\frac{1}{\tilde{\beta}} + \tilde{\phi}\right)\rho - 2\right] (\equiv \underline{r})$$

Thus, we can say that if $\underline{r}$ is sufficiently low, r_2^* satisfies $r_2^* > \underline{r}$. Therefore, $B^* < 0$, which yields the steady state related to r_2^*, showing local indeterminacy. To obtain $B^* < 0$, it is sufficient that $\underline{r} < 0$, which yields $b < 2 - \left(\frac{1}{\tilde{\beta}} + \tilde{\phi}\right)\rho$. Using the usual parameters $\{b,\ \rho,\ \beta\} = \{0.25,\ 0.05,\ 0.33\}$, this condition becomes as follows:

$$\phi > 0.028, \qquad (40)$$

---

[10] $b > \rho$ is also the necessary condition for the usual Uzawa-Lucas model (the one without labor-leisure choice) to obtain a positive long-run growth steady state.

which shows that the multiple steady states under the restriction $0.2727.. > \phi > 0.25$ derived in Equation (35), have a steady state with indeterminacy related to $r^* = r_2$ covered by Equation (40). Thus, we can conclude the following:

Result 3 *Under a sufficiently small* $\boldsymbol{\phi}(< \boldsymbol{\beta})$*, and a middle range of* $\boldsymbol{b}$*, the Uzawa-Lucas model with labor-leisure choice will yield multiple steady states with both saddle stability and local indeterminacy, and therefore global indeterminacy, under a plausible parameter set.*

While Ladrón-de-Gurevara et al. (1999) concluded that the domain of indeterminacy is narrow with regard to a production parameter (β in our model) around $\theta = 1$, we also find that the domain of indeterminacy is narrow with regard to the industriousness parameter (ϕ) for $\theta = 1$ and the usual production parameter $\beta = 0.33$.

Next, we try to find a nonzero ν, given the values $\{\beta, b, \rho\} = \{0.33..,0.25,0.05\}$. For the interval $\nu \in [-0.04,0.06]$, $\frac{\rho}{b+\nu+\rho} < \rho/b(= 0.20)$ always holds. Under $\underline{\phi} > 0.2$, the condition $\phi > \max\left[\frac{\rho}{b}, \frac{\rho}{b+\nu+\rho}, \underline{\phi}\right]$, given in Equation (27) becomes $\phi > \underline{\phi}$, which implies that the upper limit value is $\underline{\phi}$. Table VI shows that a larger ν implies a larger $\underline{\phi}$; thus, increased population growth, capital depreciation, and a positive spillover on education increase the likelihood of local and global indeterminacy.

Thus, an economy that starts to develop as a consequence of increasing industriousness always and immediately enters a domain that yields local and global indeterminacy. This domain becomes broader with a higher population growth rate and a higher spillover of population growth into human capital accumulation.

Table VI. Values of $\underline{\phi}$ for ν.

ν	$\underline{\phi}$
0.06	0.220
0.05	0.227
0.04	0.235
0.03	0.243
0.02	0.252
0.01	0.262
0.00	0.271
−0.01	0.262
−0.02	0.294
−0.03	0.306
−0.04	0.319

5. Conclusion

This study is the first step toward illuminating the relationship between industriousness and economic growth from a theoretical perspective. We apply the Uzawa-Lucas model with labor-leisure choice to the transition from no growth to long-run positive growth. The results show that industriousness may be an important factor contributing to accelerating this economic event.

The results for the dynamic paths are summarized as follows. First, economies with a lower consumption-utility share (and an equivalently larger leisure-utility share) and sufficiently low educational efficiency are stuck in a steady state with long-run zero growth. Meanwhile, a sufficiently cultivated labor supply propensity prompts the economy to grow through human capital accumulation. This finding shows how industriousness paves the way for modern economic growth.

Next, countries with mid-level efficiency of education and a mid-range utility share of leisure (and therefore of consumption) have two steady states, of which the one with the lower growth rate shows local indeterminacy under plausible parameters. As both economic paths converge toward these two steady states and can be selectable, the economic path contains global indeterminacy. This makes it difficult to select an economic path based on expectations. Since this multiplicity disappears when educational efficiency is sufficiently high, we can also conclude that economies in the early stages of economic growth have inherent fragility throughout successive periods of economic growth.

Therefore, further studies should be conducted in this area. First, our study is theoretical; thus, numerical research is insufficient. For example, the multiplicity of steady states makes it implicitly possible to yield nonlinear dynamics, as Fiaschi and Lavezzi (2007) have shown. It would be interesting to replicate nonlinear dynamics using the Uzawa-Lucas model with an endogenous labor supply, reflecting the change in attitudes toward labor from an early stage of economic growth to the modern context.

Second, there is insufficient research on the properties of the utility weight ϕ, which is the key parameter in our study. Future research should investigate the relationship between industriousness and mass consumption, which is another aspect of industriousness. The rise of the mass consumer society has not been sufficiently studied, with one important exception: a study by Matsuyama (2002). Research in this field would allow us to endogenize the dynamics of ϕ, while the present study provides a comparative analysis of just two cases of high and low ϕ. Such research would be relevant to the broader literature. In the U.S., the most successful capitalist nation worldwide to date, people work harder, even sacrificing their leisure time (see e.g., Schor 1992). This situation may be due to the coexistence of increased labor productivity and utility share, and their influence on consumption, as compared with leisure. Although Keynes (1930) predicted that increasing labor productivity would lead to a decrease in labor time, labor time has not decreased. People work longer and, therefore, spend less time on leisure, implying that modernization leads to an increase in labor share.

Third, the modern agenda for developing countries sometimes includes poverty, which makes people want to work, which also means that they are industrious. However, many people in such countries are unemployed. In the early stages, modern advanced economies had many unemployed workers, a "relative surplus-population" (Marx 1867, Ch.23), or workers supplied by the "subsistence sector" (Lewis 1954). As our model does not include unemployment, future studies should examine the relationship between industriousness and unemployment.

Lastly, our empirical results show that, in the long run, decreasing labor time trends can be observed in many countries (see e.g., Jones 2016). Our results imply that a decreasing labor supply may generate instability in an economic path derived from a multiplicity of economic pathways. Some contemporary economists are pessimistic about the risk of stagnation in the global economy. For example, Summers (2014) refers to it as "secular stagnation." As stagnation in many advanced countries has not yet been analyzed fully or clarified perfectly, it would be worthwhile to investigate this issue from the perspective of labor supply changes.

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