Decreasing Labor Supply, R&D-Based Growth, and Instability of Economic Dynamics

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Abstract

Working time has gradually decreased in the last few decades, along with the continuing growth of advanced economies. Furthermore, there have been some empirical evidence showing emerging economies' and long-run experiences of advanced economies' decreasing labor in spite of restrictions on labor statistics.

To replicate these phenomena, we develop the model with endogenous technogical chnage and endogenous labor supply, and we find that adding increasing returns of R&D (reasech and development) efficiency, at least for the small input, yields the economic path accompany the decreasing labor supply. Furthermore, the path is stable (not saddle stable), so the steady state has multiple paths under rational expectations, which yields local indeterminacy. This would reflect the modern intermittently-coming economic shocks in both advanced and emerging countries. Furthermore, the model also contains a steady state with no growth trap, and selection among steady states is possible. The model has global indeterminacy, which would be one of the mechanisms for the start of economic growth.

Keywords: Decreasing labor supply; Unstable economic dynamics; Multiple steady state; externality of R&D efficiency.

1. Introduction

Despite the continuing growth of advanced economies, working time has been decreasing gradually since 1955. Fig. 1 illustrates the average annual hours actually worked per worker in different countries from 1955 to 2020. From this figure, we observe that OECD and European Union 27 countries are shortening working time; this is especially the case for Japan, where there has been a significant decline in working time. Moreover, the average annual working hours per labor force is also decreasing, as depicted in Fig. 2. These results were also found by Aguiar and Hurst (2007), who noted that “there is a dramatic increase in leisure time that lies behind the relatively stable number of market hours worked (per working-age adult) between 1965 and 2003.” Note that increases in leisure time directly imply decreases in working time. Therefore, modern economies face a trend towards a reduction in working time.

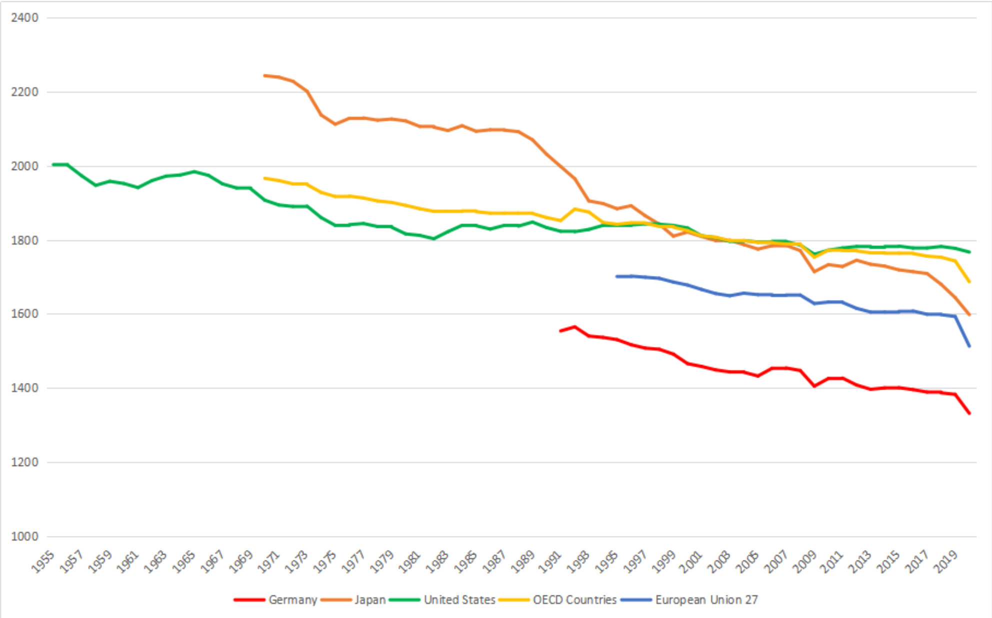

Figure 1. Average annual hours actually worked per worker (Data Source: OECD Stat.)

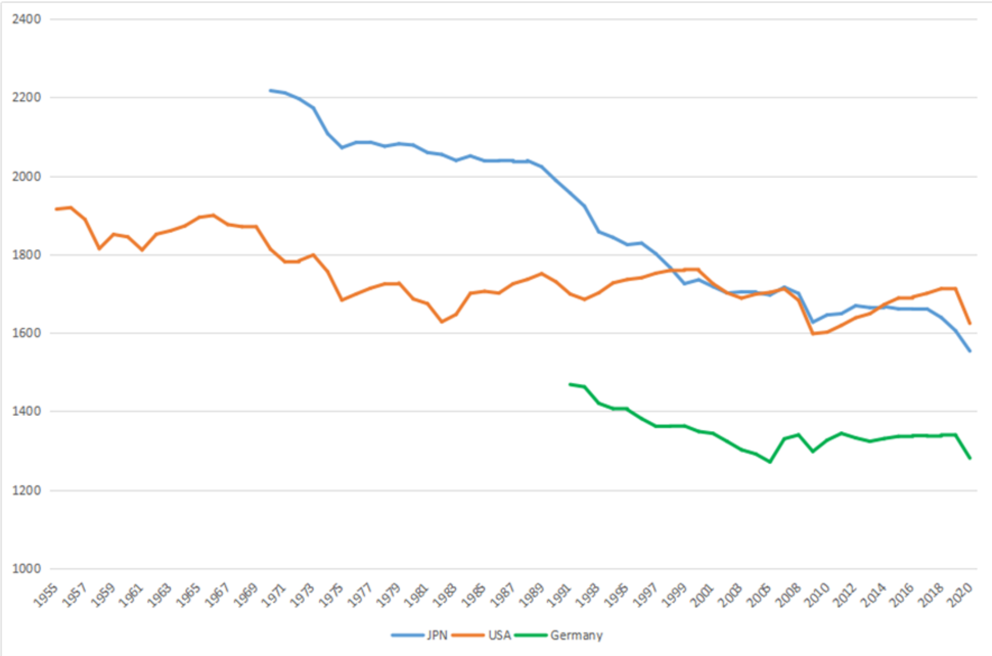

Figure.2 Average annual working hours per labor force (Data Source: OECD Stat and OECD Data.)

During this half century, many advanced economies, such as Japan and West Germany, suffered from several energy or financial crises, which shifted their economies from high-growth rates to a low and steady rate. Specifically, Japan has been stuck in severe no growth trapsover aquarter century after experiencing a temporary bubble boom. Additionally, the world economy has also been hit by several big economic shocks, such as the 2001 dotcom bubble corruption and the financial crisis of 2007–2008, between economic booms.

Alternatively, Hobara and Kuwahara (2022) collected data on the increasing working time during the early economic growth process and presented an economic model that shows that a sufficient labor supply is necessary for the start of economic growth. It is accompanied by an increase in working hours to get out of the no-growth trap. Furthermore, modern developing countries have already entered the stage of decreasing labor. For example, Bick et al. (2018) also reported the decreasing trend of labor time in developing countries. Generally speaking, both the early stage of economic growth in developed countries and modern developing countries at the early stage of economic growth have obscure timelines, making it increasingly difficult for direct comparison. However, we can conclude the following: the early stage of economic growth is characterized by a long labor hour, and that labor hour decreases throughout the process of growth.

Parallel to this phenomenon of decreasing labor hours, the whole world confronted intermittently big economic shocks and stagnation of economies (see Blanchard (2011) and Summers (2014) for an example of pessimistic perspective on the global economy.)

Is there any relationship among the decreasing working hours, the instability of economic growth, and the end of high growth for developed countries? Furthermore, is there any relationship between decreasing working hours and the start of growth or economic crisis of developed countries? To answer these questions, we develop an endogenous growth model with endogenous labor supply and further introduce R&D (research and development) externality into R&D efficiency.

Owing to the application of endogenous labor supply to the endogenous growth model, we can refer to Acemoglu and Restrepo (2018). However, in their model, researchers were used only as R&D input; furthermore, an allocation of researchers between the two types of R&D activities exists. Thus, R&D activities are always executed, and as a result, no growth taps cannot be analyzed. Therefore, the advantage of our model is to analyze no growth traps.

The Romer (1990) model is a representative model that provides an endogenous mechanism by the long-run growth and labor endowment, which is used as an R&D input. Therefore, we can connect labor supply and long-run growth rate by using our model of the variety-expansion model with the endogenous R&D input factor supply. Furthermore, linearity between researcher as the R&D input and innovation captured by newly-developed goods yield constant, non-decreasing R&D productivity. Therefore, infinite R&D productivity under zero R&D input, which inevitably yields non-zero R&D activities, can be avoided. This property enables the analysis of the emergence of no growth traps and thus escape from the traps.

As the models follow the Romer type variety expansion model, we can refer to Jones (1995), with population growth and Jones technology, and Kuwahara (2019), with capital R&D input, among others. Nonetheless, Jones technology, advocated in Jones (1995), assumes the property of *á la* Inada conditions on the R&D function, and in some research, exogenous human resource accumulation is replaced by endogenous labor supply (for example, Arnold 1998); however, this *á la* Inada condition stops growth traps from occuring. Although Kuwahara (2019) shares the assumption of increasing returns of R&D (therefore these *á la* Inada conditions lack) capital was assumed as R&D input. Thus, the decreasing labor supply was not analyzed. The basic model, the Romer model with endogenous labor supply and log-linear utility, which we develop, shows only unique, steady, balanced growth path. Therefore, for the dynamics insistent with the observed economic dynamics, we need to introduce at least one factor, thus increasing R&D returns on the at least and lower R&D investments.

As Jaffe (1986), Bernstein and Nadiri (1988, 1989), Arthur (1989), and among others, have emphasized the existence of increasing returns in innovation, we broadly consider the spillover effects on R&D activities. The Romer model treats the existing knowledge as a free stock factor on R&D but does not include increasing returns in R&D activities. Using the R&D spillover effects, some theoretical studies, such as Chen and Chu (2010) and Kuwahara (2019), have developed a model with increasing returns and obtained multiple steady states. However, they do not include the decreasing labor supply.

The strength of this model is the connection between empirically observed labor supply and growth patterns. The aim of this research, namely, the development of an integrated model that generates two phenomena—decreasing labor supply and instability of economic paths—we assume the spillover of technology. A necessary assumption is the property that for a lower R&D input, the R&D efficiency is lower, which is considered to reflect the externality effects of R&D. This assumption generates three possible equilibrium labor allocations, and these equilibriums correspond with high, low, and zero growth steady states, respectively. Owing to the fact that the labor allocation is jumpable,

and the selection of steady states depends on this labor allocation, if some large shock hits the economy, the expectations might change, as well as the selected steady states.
Low equilibrium is the stable equilibrium with zero growth rate and without transition path. Thus, the economy caught in this equilibrium cannot escape from this if the expectations are not changed.
The middle equilibrium has multiple uncountable infinite paths on a stable manifold. Therefore, the steady state has a locally indeterminacy and formation of rational expectation that cannot yield a unique dynamic path. Thus, the economy could emerge as unstable because of the difficulty in forming expectations.
Finally, we emphasize that our model is insistent on the phenomena of long time labor in the early stage of economic growth, as reported by Bick et al. (2018), and Hobara and Kuwahara (2022). Our model shows that a sufficiently small labor supply yields a no-growth steady state, and this equilibrium can be one of multiple steady states that contain positive growth. In this case, the start of economic growth in developing countries are explained by the selection of optimistic expectations

The rest of the study is organized as follows: Section 2, establishes the basic Romer model that contains labor-leisure choice; and Section 3, shows the existence of the unique stable steady state under the usual Romer setting with a constant R&D parameter. In Section 4, we introduce the spillover of R&D and derive a multiple steady states containing a path with decreasing labor supply. Finally, Section 5 concludes the study.

2. The Model

To analyze the relationship between growth dynamics and (decreasing) labor supply, we utilize the simple Romer model, which is eliminated with capital accumulation; however, equips endogenous labor supply. Household is assumed to be a representative one, and final goods are used as numéraire.

2.1. Household

The economy admits a continuous unit measure for household. A worker in the representative household derives utility from consumption while deriving disutility from labor supply.
To simplify the dynamics, we use a specific utility function as

$$U(C,L) = \ln\ C - \frac{L^{1+\chi} - 1}{1+\chi}, \tag{1}$$

where C, L, and χ denote consumption, labor supply, and the inverse of the Frisch elasticity of labor supply, respectively, which captures the elasticity of hours worked to the wage rate, given a constant marginal utility of consumption. Notably, this function form is essentially the same as the one used in various studies, such as Acemoglu and Restrepo (2018), and it ensures the convexity of the utility.
Given the interest rate r and wage rate w, workers choose consumption and labor supply to maximize their lifetime utility

$$\int_0^\infty e^{-\rho t}\ U(C(t), L(t))\mathrm{d}t,$$

subject to the resource constraint $C(t) + \dot{W}(t) \leq r(t)W(t) + w(t)L(t)$, where ρ is the rate of time preference, and W is the asset holdings (wealth), which is composed by equity of the R&D firms (or say, total monopoly profit) in this study. This optimization problem yields the usual Euler equation given by

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho, \tag{2}$$

and labor supply satisfies

$$L(t)^{\chi} = \frac{w(t)}{C(t)}. \tag{3}$$

2.2. Goods Production

The supply side contains three sectors: final goods production, intermediate goods production, and research sector; and this section treats the former two sectors. The equilibrium is determined by instantaneous optimization. Thus, we omit the time index in this subsection.

The final goods are assumed to be used by consumption or intermediate input, and the production is assumed to be perfectly competitive. Firms in this sector produce a unique final good, Y, by hiring labor and using various intermediate goods. The aggregate production function is specified as follows.

$$Y = L_Y^{1-\alpha} \int_0^A \tilde{X}\,(i)^{\alpha} di, \tag{4}$$

where L_Y denotes the labor who produces final goods, $\tilde{X}$ denotes the ith intermediate goods input, and A denotes the variety of intermediates. As shown below, greater varieties of intermediate goods help increase the final goods. Thus, an increase in A represents the technological process.

This study chooses the price of the final goods as the numeraire. Therefore, wages and the price of intermediates are given by

$$w = (1-\alpha) L_Y^{-\alpha} \int_0^A \tilde{X}\,(i)^{\alpha} di, \;\; p(i) = \alpha L_Y^{1-\alpha} \tilde{X}(i)^{\alpha-1}, \tag{5}$$

where $p(i)$ is the price of i-th intermediate good.

Intermediate goods can be considered as, specific machines, and are assumed to be supplied by monopolists who hold relevant patents. Therefore, intermediate goods are supplied monopolistically. Additionally, a firm with a patent for ith intermediate goods production can be designated as an ith intermediate goods firm. It is assumed that producing one unit of intermediate goods requires η units of final goods. Subsequently, the profit of an ith intermediate good firm is given as

$$\tilde{\Pi}(i) = p(i)\tilde{X}(i) - \eta \tilde{X}(i). \tag{6}$$

The intermediate goods firm maximizes this profit subject to the iso-elastic demand curves as in Eq. (5), then, the price of an intermediate is a constant markup over marginal cost

$$p(i) = \frac{\eta}{\alpha} (\equiv p). \tag{7}$$

Subsequently, the intermediate goods supply and profits of the ith firm are as follows:

$$\tilde{X}(i) = \left(\frac{\alpha^2}{\eta}\right)^{\frac{1}{1-\alpha}} L_Y \equiv \tilde{X} \tag{8}$$

$$\tilde{\Pi}(i) = \frac{1-\alpha}{\alpha}\eta\tilde{X} = (1-\alpha)\alpha\left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}} L_Y \equiv \tilde{\Pi}. \tag{9}$$

It should be noted that these imply symmetric intermediate firms.

2.3. R&D Activities

The R&D sector is assumed to be free-entry R&D firms, who are members of this sector until their profits become zero. Firms act to create the designs of new intermediate goods by using labor, and obtaining the perpetual patents for newly developed designs, which bear the perpetual stream of monopoly profits $\tilde{\Pi}$. The evolution of innovations is assumed to be the usual Romer (1990) type as follows:

$$\dot{A}(t) = \delta A(t) L_A(t), \tag{10}$$

where δ denotes the productivity of research firms and L_A denotes the labor working in the research firms. Although the usual Romer model assumes a constant R&D parameter, here, we allow the change of R&D parameter δ, which captures the spillover of R&D activities. Same as the usual Romer model, we assume that the R&D activities use the entire knowledge capital stock A as existing variety; furthermore, we assume that R&D efficiency is a variable that reflects the instantaneous vigorousness (or flow) of R&D activities.

The R&D is executed by only employment of labor (as researchers), and the obtained economic value per one developed variety is denoted as $\tilde{V}$, which is the sum of present value of unit patent given as

$$\tilde{V}(i,t) = \int_t^\infty \tilde{\Pi}\,(i,\tau) e^{-\int_0^\tau r(s)ds} d\tau, \tag{11}$$

which is given for a created innovation, creation of new innovation follows Eq. (10). We assume symmetric equilibrium; thus, we can omit variety index i. Time difference of Eq. (11) immediately yields the asset equation, given as follows:

$$r\tilde{V}(t) = \dot{\tilde{V}}(t) + \tilde{\Pi}(t). \tag{12}$$

Subsequently, the profits of the jth R&D firm, Π_j^{RD}, are given by

$$\Pi_j^{RD}(t) = \dot{A}_j(t)\tilde{V}(t) - wL_{Aj}(t) = (\delta A(t)\tilde{V}(t) - w(t))L_{Aj}(t) \tag{13}$$

where $\dot{A}_j$ and L_{Aj} denote the number of developed intermediate by the jth firm and labor (researcher) input on R&D by the jth firm, respectively. The second equation uses Eq. (10). Therefore, the free entry of the R&D activities yields the zero profit condition on R&D firm, that is,

$$L_A(t) = 0 \text{ and } w(t) > \delta A(t)\tilde{V}(t), \quad \text{or} \quad L_A(t) > 0 \text{ and } w(t) = \delta A(t)\tilde{V}(t). \tag{14}$$

The former conditions are related to the case without R&D, and the latter conditions are related to the case with R&D.

2.4. Aggregation Values and their Dynamics

In this section, we derive aggregate values. Considering that all labors are allocated to the

production of R&D sectors, we assume $u(t) = L_Y(t)/L(t)$ to be the fraction of production workers and $1 - u(t) = L_A(t)/L(t)$ to be that of R&D workers.
Aggregating (8) yields the aggregate input of final goods to intermediate X:

$$X(t) \equiv \int_0^{A(t)} \eta \tilde{X}(t) di = \alpha^2 \left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}} u(t)A(t)L(t), \tag{15}$$

where η stems from the necessary unit of intermediate production. Using Eqs (4) and (??0), we obtained aggregate output Y and the aggregate spending on intermediate goods X on the presentation of using u, A, and L as follows:

$$Y(t) = \left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}} u(t)A(t)L(t), \quad \text{and} \quad X(t) = \alpha^2 Y(t). \tag{16}$$

Uniting the above equations and the resource constrain on the final goods ($Y = C + X$), we have

$$C(t) = (1-\alpha^2)\left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}} u(t)A(t)L(t) = (1-\alpha^2)Y(t). \tag{17}$$

The above three equations indicate that the constant rate of the final goods are used by consumption and intermediate goods input.
Here, we add Eqs. (9) and (??0). Subsequently, the per variety monopoly profit $\tilde{\Pi}$ and total monopoly profit Π can be derived as follows:

$$\tilde{\Pi}(t) = (1-\alpha)\alpha\left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}} u(t)L(t) = (1-\alpha)\alpha\frac{Y(t)}{A(t)}, \tag{18}$$

$$\Pi(t) = \int_0^A \tilde{\Pi}(t)\, di = (1-\alpha)\alpha\left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}} u(t)A(t)L(t) = (1-\alpha)\alpha Y(t), \tag{19}$$

Further, equation (5) can be rewritten as

$$w(t) = (1-\alpha)\left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}} A(t)\left(= (1-\alpha)\frac{Y(t)}{u(t)L(t)}\right), \tag{20}$$

Here, we derived some important relationships used below, as follows: Firstly, by substituting (17) and (20) into (3), we can obtain

$$L(t)^{1+\chi} = \frac{1}{1+\alpha} u(t)^{-1}. \tag{21}$$

Secondly, using Eqs. (10), (21), and (17), Eq. (2) is transformed as follows:

$$-\chi\frac{\dot{L}(t)}{L(t)} + \tilde{\delta}\left(1 - \frac{1}{1+\alpha}L(t)^{-1-\chi}\right)L(t) = r(t) - \rho. \tag{22}$$

3. Dynamics and Steady States under a Constant R&D Efficiency

Here, we use the model described in the previous section to analyze the dynamics of the economy. In this section, we started with a simple case where there were R&D activities with a constant and positive efficiency; later, we discussed the case where there were no

R&D activities.

In this section, we first analyzed the existence of a steady-state with a usually-assumed constant R&D efficiency (namely, we provisionally assume $\delta(t) = \tilde{\delta}(constant)$). Subsequently, we derived the conditions for the steady-state. From Eqs. (10) and (14), the economy is in equilibrium only if $\tilde{V} = w/(\tilde{\delta}A)$ for $L_A > 0$. Later, we will discuss the case of no R&D activities, i.e., $L_A = 0$.

3.1. Steady State with R&D

Suppose that R&D firms hire labor to generate innovations with a constant and positive efficiency, that is, $L_A > 0$ and $\delta = \tilde{\delta} > 0$ is constant. Then, from Eqs. (10) and (14), the economy is in equilibrium only if $\tilde{V} = w/(\tilde{\delta}A)$ for $L_A = (1-u)L > 0$. Using Eq. (20) yields

$$\tilde{V}(t) = \frac{w(t)}{\tilde{\delta}A(t)} = \frac{1-\alpha}{\delta}\left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}}. \tag{23}$$

This implies that $\dot{\tilde{V}} = 0$ in equilibrium and the assumption of $\delta(t) = \tilde{\delta}$. Then, substituting Eqs. (18), (21), (23), and $L_Y = uL$ into Eq. (12), the interest rate is given by

$$r(t) = \frac{\tilde{\Pi}(t)}{\tilde{V}(t)} = \alpha\tilde{\delta}(1+\alpha)L(t)^{-\chi}. \tag{24}$$

Notably, in spite of constant $\tilde{V}$, the interest rate r may change corresponding to the change of labor allocation.

Substituting Eq. (24) into the equation Eq. (22), we obtain the dynamical equation on L as follows:

$$\chi\frac{\dot{L}(t)}{L(t)} = \tilde{\delta}\,\underbrace{(L(t) - L(t)^{-\chi})}_{\equiv\Psi(L(t))} + \rho. \tag{25}$$

As $\Psi(1) = 0$, $\lim_{L\to 0}\Psi(L) = -\infty$, and $\Psi'(L|L>0) > 0$, there exists a unique $L^* \in (0,1)$ such that $\Psi(L^*) + \rho = 0$, and thus, we obtain Fig.3.

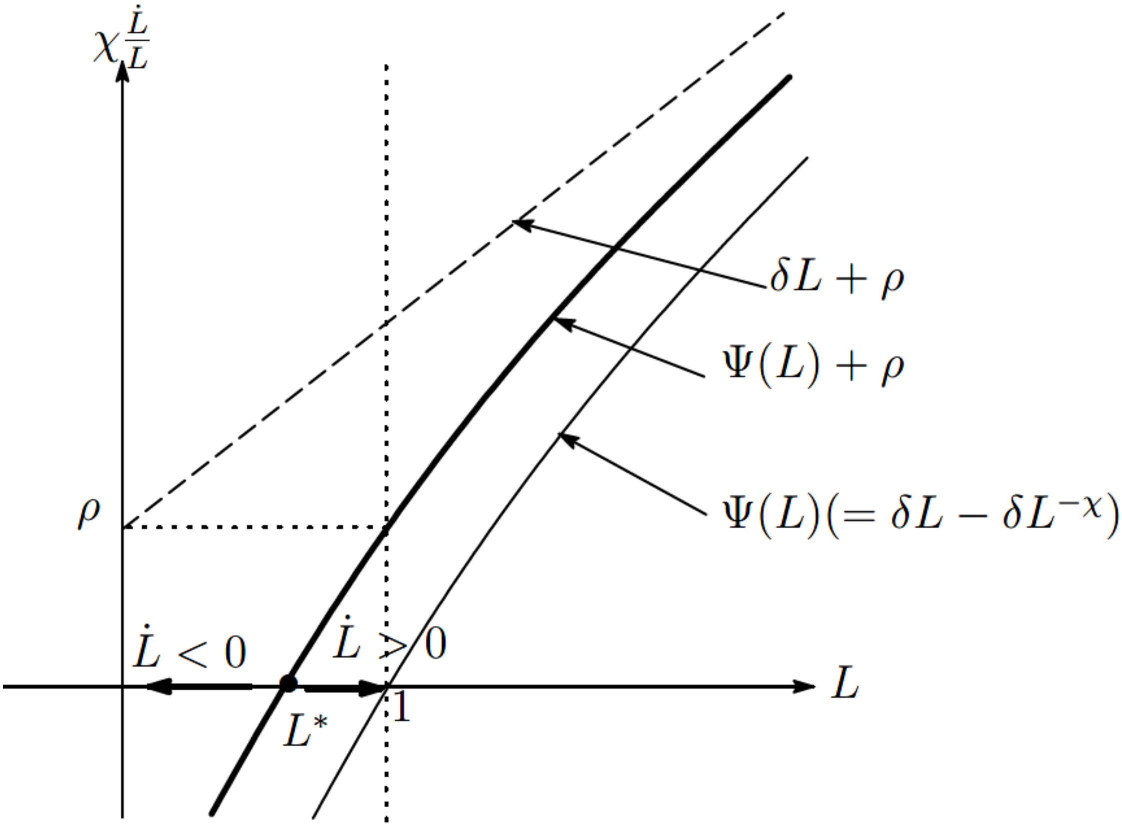

Figure 3. Dynamics of L

From the properties of the dynamics of L depicted in Fig. 3, we observed that the unique equilibrium L^* is unstable. Furthermore, $L=0$ makes the marginal utility of consumption infinite, whereas $L=1$ makes leisure infinite; therefore, the perpetually sustainable economic path under rational expectations is uniquely determined, and it is one that selects L^* at inital time, and permanently stays there. Thus, (under the satisfaction of all feasible conditions) the economy on the steady state with positive R&D has a unique balanced growth path (BGP), and along the BGP, the economy growth rate is given by

$$g^* \equiv \frac{\dot{Y}}{Y} = \tilde{\delta}\underbrace{L^*\left(1-\frac{1}{1+\alpha}L^{*-(1+\chi)}\right)}_{\equiv L_A{}^*}. \tag{26}$$

Notably, it is necessary to examine whether L^* satisfies the positive R&D activities, that is, $L_A{}^* > 0$. Conversely, when $L_A = 0$, there are no R&D activities (that is, $\dot{A} = 0$), and all workers participate in the production sector (that is, $u = 1$). The conditions of the steady state with or without R&D are discussed in the subsequent sections.

Here, we examine the conditions for positive R&D equilibrium. For the L^* obtained above to be at equilibrium, it must satisfy the feasible condition on L^* and u^*(or $L_A{}^* \in (0, L^*)$). Equation (21) implies $L > 0$ yields $u > 0$, and $u < 1$ requests $L > \underline{L} \equiv (1+\alpha)^{-\frac{1}{1+\chi}}$. $\alpha > 0$ and $\chi > 0$ satisfy $\underline{L} \in (0,1)$. Thus, $L \in (\underline{L}, 1)$ always exist, and this is the necessary condition for a feasible L^*.

From the function form of $\Psi(L)+\rho$, this condition yields the following two conditions; $\Psi(1)+\rho>0$ and $\Psi(\underline{L})+\rho<0$. The latter is trivial because $\Psi(1)+\rho=\rho>0$, and the former yields

$$\tilde{\delta}>\frac{\rho}{\chi}(1+\alpha)^{\frac{1}{1+\chi}}(\equiv\underline{\delta}). \tag{27}$$

This condition indicates that a sufficiently large R&D parameter is necessary for the existence of a positive R&D steady state. A small ρ and α, and large χ minimizes $\underline{\delta}$; consequently, an economy with a lower δ has the capability to grow.

3.2. Steady State without Positive R&D

The previous sections discussed the steady-state equilibrium with positive R&D. In contrast, in this section, we assume $u^{**}=1$, namely $L_Y^{**}=L^{**}$, and therefore, an economy caught by a no growth trap $g^{**}=0$.

In this steady state, $g_A^{**}=0$, and Eq. (2) imply $r^{**}=\rho$. Therefore, $\tilde{\Pi}$ in Eq. (18) is constant in the steady state, and therefore, $\tilde{V}^{**}$ in Eq. (23) is constant (namely $\dot{\tilde{V}}=0$).

Under satisfying the above conditions, the condition of Eq. (14) must hold. Thus, we obtained the condition as follows:

$$\tilde{V}^{**}=\frac{(1-\alpha)\alpha}{\rho}\left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}}L^{**}<\frac{1-\alpha}{\tilde{\delta}}\left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}}. \tag{28}$$

This condition is made into

$$L^{**}<\bar{L}\equiv\frac{\rho}{\alpha\tilde{\delta}}. \tag{29}$$

From Eq. (21), optimal labor supply of labor is given as $L^{**}=\underline{L}$. Therefore, the necessary condition of existence of no growth traps is given as $\underline{L}<\bar{L}$, which is made into

$$(1+\alpha)\left(\frac{\rho}{\alpha\tilde{\delta}}\right)^{1+\chi}>1. \tag{30}$$

A large α, ε, ρ and a small $\tilde{\delta}$ yields the possibility of no growth traps. By solving this condition of $\tilde{\delta}$, we obtained

$$\tilde{\delta}<\frac{\rho}{\alpha}\{(1+\alpha)\}^{\frac{1}{1+\chi}}(\equiv\bar{\delta}) \tag{31}$$

This condition shows that if $\tilde{\delta}$ is sufficiently small, then $u=1$ and $\dot{A}=0$ are realized, which imply that all workers enter the production sector and R&D activities stop. As $\dot{Y}/Y=\dot{A}/A=0$, economy stagnation occurs.

, Ă-, é• B Thus, we obtained the condition for a long run positive growth given in Eq. (27) and that for no growth in Eq. (31), and we have the following result:

Lemma

Under the usual R&D structure, non-existence of increasing return of R&D or spillover of R&D activity, Eq. (27) and (31) imply that the multiple steady states do not emerge under R&D with labor supply.

A larger $\tilde{\delta}$ and a smaller α and ρ, namely smaller endurance for labor and time, yields a positive steady economic growth, and lack of them ensures no growth traps. This property follows the basic existing results of the Romer model.

This follows the next proposition.

Proposition 1

The economy with endogenous labor supply and with spillover of R&D activities has a unique steady-state equilibrium if and only if R&D efficiency is sufficiently large. Otherwise, if R&D efficiency is not sufficiently large, then all workers enter the production sector and economy stagnation occurs.

So far, our basic arrangement of log-liner utility function, no capital accumulation, and the Romer type R&D efficiency, yields only the usual result that a high R&D efficiency is necessary for long-run positive growth, and there is no transition dynamics, despite the endogenous labor supply and endogenous decision on R&D activities. Therefore, we extend this basic model by introducing the least additional factor, that is, the R&D spillover.

4. Dynamics and Steady States with Threshold Externality

The previous section has established the existence of a BGP with positive R&D activities. However, as broadly recognized, R&D activities contain various positive spillover effects. In this study, we assume that these effects shift the R&D efficiency, and then discuss how does the externality affects the economy.
Em à la Inada Conditions, namely, if infinite large marginal efficiency for the R&D input tending tozero, then, the equilibrium without R&D activities cannot exist. This condition directly contradicts the main concern in this study: decreasing labor supply and comparative stagnation of the economic performance of developed countries. As our main aim is not to analyze the determination of a long-run growth rate, our critical assumption is that ϕ is an increasing function for a sufficiently small R&D input, and it is simply constant under a sufficiently large R&D input.

4.1. R&D Efficiency

In contrast to the constant R&D parameter in the Romer (1990) model, the R&D efficiency varies in the Jones (1995) model, the so-called Jones technology, but the effects are completely opposite for the aim of the present study. As it has an infinitely large marginal efficiency for the R&D input, which tends to zero, the equilibrium without R&D activities cannot exist. The main concern in this study is the decreasing labor supply and comparatively low growth of developed countries, and not the determination of the long-run growth rate. Thus, we assume that R&D efficiency is an increasing function for a sufficiently small R&D input and that it is simply constant for a sufficiently large R&D input. Here, to simplify the expression, following Kuwahara (2019), we impose the next assumption.

Assumption 1.

R&D efficiency is a function of the number of researches with threshold Λ, that is,

$$\delta = \begin{cases} \tilde{\delta}, & \text{if} \quad L_A \geq \Lambda \\ \phi(L_A), & \text{if} \quad L_A \in (0, \Lambda), \\ 0, & \text{if} \quad L_A = 0 \end{cases} \tag{32}$$

where $\tilde{\delta} > \bar{\delta}$ *is constant, and* $\phi(\cdot)$ *is an increasing function satisfying* $\lim_{L_A \to \Lambda} \phi(L_A) = \tilde{\delta}$*, and* $\lim_{L_A \to 0} \phi(L_A) = 0$.

That is, as Figure 4 illustrates, R&D efficiency increases with the scale of inputs if the scale is smaller than the threshold Λ, and is constant otherwise. Thus, marginal R&D efficiency is smaller in near zero R&D input.

Specification:

In particular, we set $\phi(\cdot)$ *as*

$$\phi(L_A) = \frac{\tilde{\delta}}{\Lambda} L_A \equiv \varphi L_A. \tag{33}$$

The specification of $\phi(L_A)$ is described in Fig.4.

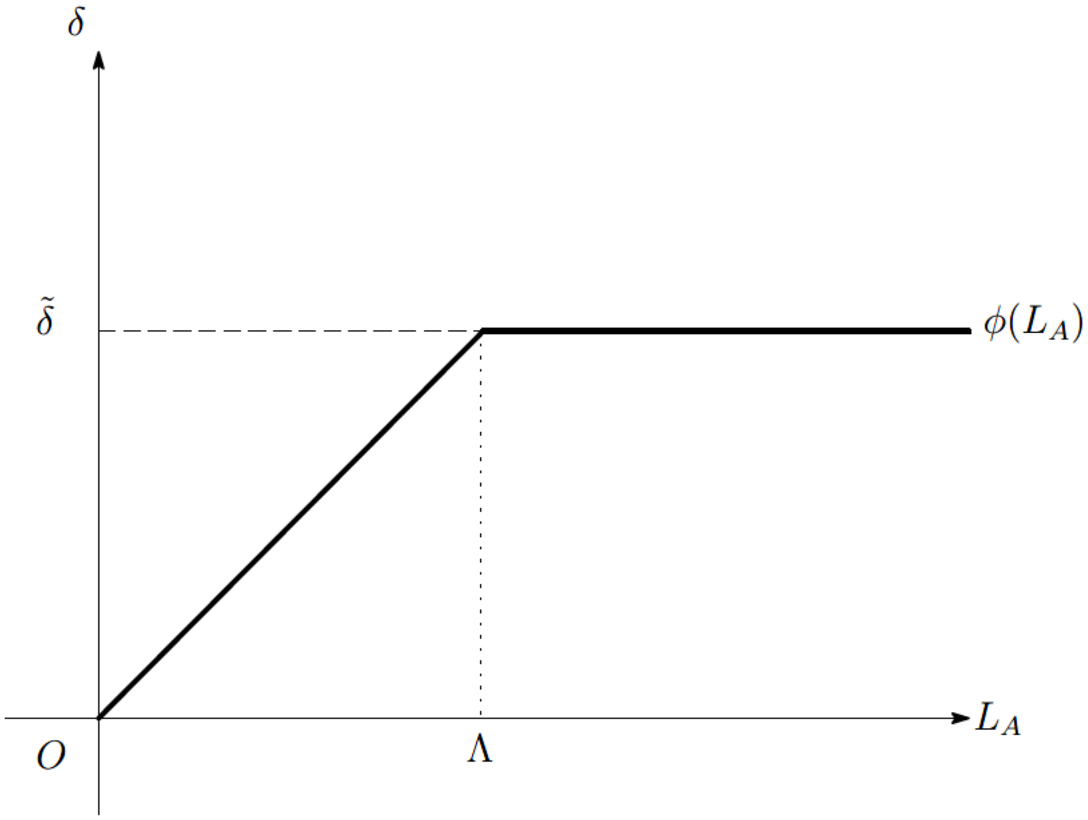

Figure 4. Specification of δ

Furthermore, for simplicity, we assume that the threshold value, which generates or does not spillover from R&D, is sufficiently small as follows: $\Lambda < L_A{}^*$, where $L_A{}^*$ is the R&D input in the steady state derived in Section 3. Under this assumption, the existence of a steady state related with $L_A{}^*$ is unchanged, which saves additional derivation for the extra steady state and avoids complication on analysis.

4.2. Dynamics and Steady states

Under the assumption of Eq. (32) and the specification of Eq. (33), Eqs. (21) and (25) yield

$$\rho = \alpha\varphi(1-u)uL^2 - \varphi(1-u)^2L^2,$$

$$(LHS(L) \equiv)\frac{(1+\alpha)\rho}{\varphi L^2} = (1+\alpha-L^{-1-\chi})(L^{-1-\chi}-1)(\equiv RHS(L)). \quad (34)$$

Whereas the form of the LHS is trivial, the form of the RHS is somewhat complicated. From $\lim_{L\to 0} L^{-1-\chi} = \infty$ and $\lim_{L\to\infty} L^{-1-\chi} = 0$, we obtain $\lim_{L\to 0} RHS(L) = -\infty$ and $\lim_{L\to\infty} RHS(L) = -(1+\alpha)$, and $RHS(\bar{L}) = 0$ and $RHS(\underline{L}) = 0$, where we denote two solutions of $RHS(L) = 0$ as $\bar{L}, \underline{L}$ $(\bar{L} > \underline{L})$, and from Eq. (34), these two solutions are derived as $(1+\alpha)^{-\frac{1}{\chi}}$ and 1. As $(1+\alpha)^{-\frac{1}{\chi}} < 1$, we obtain

$$\bar{L} \equiv 1 > \underline{L} \equiv (1+\alpha)^{-\frac{1}{1+\chi}}. \quad (35)$$

Furthermore, we define $\hat{L} \equiv \arg\{L|RHS'(L) = 0\}$, which is the labor L that yields the maximum value of RHS. Using the RHS, we obtain

$$RHS'(L) = (1+\chi)L^{-\chi-2}[2L^{-1-\chi} - (1+\alpha+2)], \quad (36)$$

which yields $\hat{L} = \left(1+\frac{\alpha}{2}\right)^{-\frac{1}{1+\chi}}$.

Here, we easily show $\underline{L} < \hat{L} < \bar{L}$. Uniting the above properties, we obtain Fig.5 as the graph of RHS and LHS.

Following Vissing-Jørgensen and Attanasio (2003) and Xu (2017), $IES > 1$, namely $\sigma < 1$ is realistic. Thus, we can assume $\bar{L} < 1$ to hold.

Substituting $\underline{L}$ into Eqs. (21) or (21) yields $u = 1$. This shows that $\underline{L}$ is the labor supply when the poverty traps occur.

From Eqs. (10), (33) and (21), we obtain the steady state growth rate of this economy, which is a function of L, as follows:

$$g(L) = \varphi\left(1 - \frac{L^{-1-\chi}}{1+\alpha}\right)^2 L^2 \quad (37)$$

As this is a quadratic function with positive coefficient, a larger labor supply L relates to a higher growth rate of the economy.

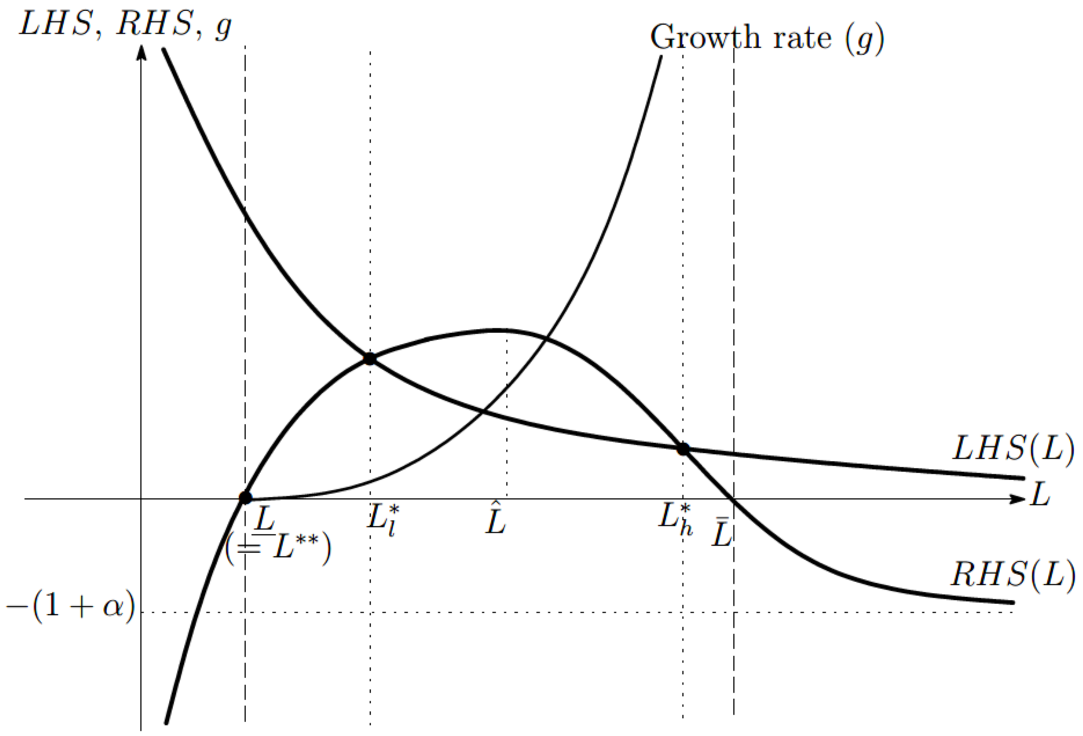

Figure 5. Steady States of $\boldsymbol{L}$

Here, we have the dynamics of L as follows:

$$\chi\frac{\dot{L}}{L} = \varphi L^2\{LHS(L) - RHS(L),\} \tag{38}$$

we find that L_l^* is stable and L is jumpable; therefore, it is a stable solution that has multiple, uncountable infinite path that converge L_l^*. Thus, the steady states related with L_l^* generate local indeterminacy. Then, L_h^* is unstable, hence the path that selects L_h^* at initial time and remains there satisfies the rational expectation equilibrium, and it is the locally stable stationary solution without transitional dynamics.

The path that chooses L_h^* at the initial time and keeps it is a reasonable expected equilibrium path and a locally stable stationary solution with no transition dynamics. As $\lim_{L\to 0}(LHS(L) - RHS(L)) = \infty$, we obtain the dynamics of L as given in Fig. 6.[1]

[1] Note that $\max_{L}\{RHS(L)\} = RHS(\hat{L}) = (1+\alpha-1-\alpha/2)(1+\alpha/2-1) = \alpha^2/4$ is smaller than $(1-\alpha)$ if and only if $\alpha < \sqrt{8}-2 \simeq 0.83$. As α is usually not significant large and $LHS(\hat{L}) > 0$, $LHS(\hat{L}) - RHS(tL) > -(1-\alpha)$ always holds.

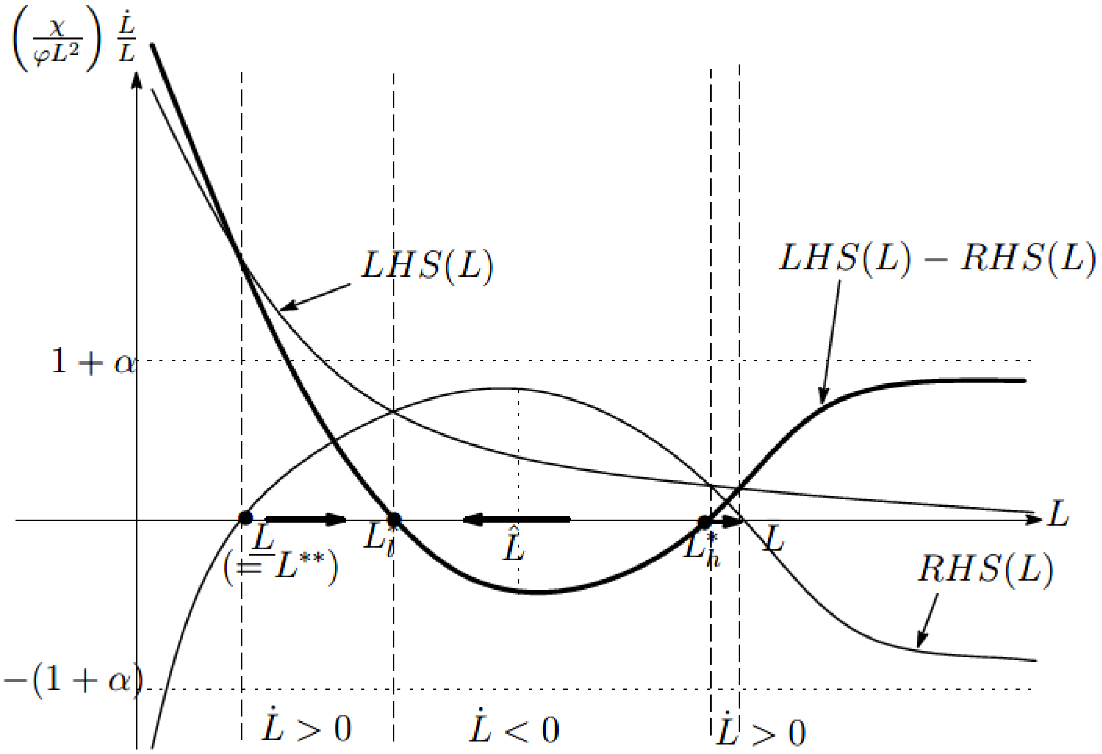

Figure 6. Dynamics of L

Figure 6 shows that high equilibrium (denoted by L_h) is a steady, balanced growth path, and low equilibrium (denoted by L_l) has instability, which is caused by local indeterminacy, which yields difficulty in forming common expectations. Furthermore, $\delta(L_A)|_{L_A=0} = 0$ yields the steady state with zero growth (denoted by $\underline{L}$) that relates to $L_A = 0$. Thus, in addition to the local indeterminacy related to the low equilibrium, our model yields global indeterminacy on selection among these three steady states: the high-level equilibrium, which is a BGP; unstable low-level equilibrium, which has a local uncertainty; and the zero-growth equilibrium with $L_A = 0$, which always exists because of $\delta(L_A)|_{L_A=0} = 0$.

These dynamical properties can be considered as the mechanism that relates to the decreasing labor hour and stagnation of economic growth, as well as the instability of the economic path. Furthermore, the model shows that underdeveloped countries (corresponding to the ones in steady states with zero growth) can escape the traps by changing the pessimistic expectations of realizing $\underline{L}$ into the optimistic ones of realizing L_h or at least L_l.

5. Conclusion

In this study, we confronted the instability in the world economic growth path, which has experienced several significant economic shocks intermittently, which implies some of the difficulty in forming expectations, namely the existence of multiplicity for economic

paths. Furthermore, we also observed the decreasing labor supply in the developed economies. We presented the model that unifies these two factors by introducing endogenous labor supply and R&D spillover into the R&D efficiency in the basic Romer model.
In this method with R&D spillover, we obtain three steady states, which are respectively related to the labor supply values; L_h^*, L_l^*, L^{**}. L_h^* has high growth rate, a large labor supply, and is unstable in the sense that an accurate expectation of L_h^* is necessary to realize this steady state. L_l^* has a low growth rate, middle labor supply, and is stable in the sense that multiple expectations of converging L_l^* are possible; therefore, deterministic expectation is impossible and the economic path fluctuates. L^{**} has a zero growth rate, small labor supply, and is unstable in the sense that an accurate expectation of L^{**} is necessary to realize this steady state.

These results are the same for both developed and developing economies. The developed economies shift their steady state from the steady state with $L_h^*$to the steady state with L_l^*. Consequently, the developed economy seems to lose the stability of their economic growth, and significant economic shocks occasionally hit them. The developing countries also have the possibility of economic growth, but a robust brief on no-growth and lower labor supply prevent them. Consequently, the economy is stuck in the steady state L^{**}. We can therefore conclude that these phenomena are roughly sketched by our model.

We have to note that our model is still restrictive. Some studies, such as Abramobiz and David (1973), indicates that the growth of American TFP shifts from capital accumulation dominance to knowledge accumulation dominance. The data of Krugman (1991) indicates that the economic development of East Asia is dominated by capital accumulation. In this study, for the sake of simplification and to focus on the decreasing labor supply, capital accumulation is abstracted and differences between developed and developing countries are not considered. Furthermore, although this study focuses on the function of labor, it does not address the issue of the labor share, which has received attention in recent years. We consider these problems for future studies.

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